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Short communication

Thermal stability conditions of a weakly interacting Fermi gas in a weak magnetic field

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Abstract

On the basis of the results derived from pseudopotential method and ensemble theory, thermal stability of a weakly interacting Fermi gas in a weak magnetic field is studied by using analytical method of thermodynamics. The exact analytical expressions of stability conditions at different temperatures are given, and the effects of interactions as well as magnetic field on the stability of the system are discussed. It is shown that there is an upper-limit magnetic field for the stability of the system at low temperatures, and there is an attractive dividing value at high temperatures. If attractive interaction is lower than the critical value, the stability of the system has no request for magnetic field, but if attractive interaction is higher than the dividing value, a lower-limit magnetic field exists for the stability of the system.

Keywords: Fermi gas; Interaction; Heat capacity; Stability

1. Introduction

In the recent years, many research results have been obtained for the investigation of the thermodynamic properties of interacting system [1–9] and of the stability of interacting system. For example, the studies in Refs. [10–13] show that interparticle interactions can make the condensate of Bose gas disintegrate and collapse; Modugno [14] found that degenerate Fermi gas may collapse under certain conditions; Yuan [15,16] investigated the stabilization of weakly interacting Bose and Fermi gases from the point of view of thermodynamics, and gave the particle number density conditions of instability of the system. However, theoretical and experimental studies on stability of Fermi system are still in the initial stages. Therefore, it is very important to investigate the effects of interactions on stability of the system under different conditions, and to give the exact conditions of stability for discussing physical properties of Fermi system.

It is well known that thermodynamics stability should involve mechanical stability and thermal stability for a thermal-equilibrium system contacted with a heat source at constant temperature and pressure. Men et al. [17] studied the mechanical stability of a weakly interacting Fermi gas in a weak magnetic field, but the thermal stability of the system was not considered. In this study, based on the heat capacity results derived from pseudopotential method and ensemble theory, the thermal stability conditions of a weakly interacting Fermi gas in a weak magnetic field are studied, the analytical expressions of thermal stability conditions at different temperatures are given, and the effects of magnetic field and interactions on thermal stability of the system are analyzed. The upper-limit magnetic field for the thermal stability of the system at

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low temperatures, the attractive critical value and the lower-limit magnetic field for the thermal stability of the system at high temperatures are also given.

2. Thermal stability conditions at different temperatures

We consider a weakly interacting system of spin-1/2 fermions, with volume V and particle number N, in a weak magnetic field $B = B_z$. According to the pseudopotential method and ensemble theory, if only spin motion is considered, the heat capacity of system at different temperatures can be written as [18]

When $T \to 0$,

$$C_v = 0 \tag{1}$$

When $T < T_F$,

$$C_v = \frac{\pi^2}{2}\frac{NkT}{T_F}\left\{1 - \frac{1}{2}\left(\frac{\mu B}{\varepsilon_F}\right)^2\left[1 + \frac{3}{2}\frac{\alpha n}{\varepsilon_F}\right]\right\} \tag{2}$$

When $T \gg T_F$,

$$C_v = \frac{3}{2}Nk\left\{1 + \frac{4}{3}\left(\frac{\mu B}{kT}\right)^2\left[1 + \frac{3\alpha n}{2kT}\right]\right\} \tag{3}$$

When $T > T_F$,

$$C_v = \frac{3Nk}{2}\left\{1 + \frac{4}{3}\left(\frac{\mu B}{kT}\right)^2\left[1 + \frac{3\alpha n}{2kT}\right] - \frac{n\lambda^3}{2^{9/2}}\left[1 + \frac{70}{3}\left(\frac{\mu B}{kT}\right)^2\right]\right\} \tag{4}$$

where C_v is the heat capacity, k is the Boltzmann constant, T_F is the Fermi temperature, ε_F is the Fermi energy, n is the density of particle number, T is the temperature of the system, μ is the Bohr magneton, $\alpha = ah^2/(\pi m)$ is the parameter of interactions, $|a|$ is the s-wave scattering length, h is the Plank constant, and m is the mass of single fermion.

When the system stays in thermodynamic equilibrium and maintains the thermal stability, it should satisfy the condition that $(\partial T/\partial S)_v = T/C_v > 0$, and $C_v > 0$ at $T > 0$, where S is the entropy of the system. From these, we can get the thermal stability conditions at different temperatures by the following calculation:

When $T < T_F$, the expression $T/C_v = [\frac{\pi^2}{2}\frac{Nk}{T_F}]^{-1}\{1 - \frac{1}{2}(\frac{\mu B}{\varepsilon_F})^2[1 + \frac{3}{2}\frac{\alpha n}{\varepsilon_F}]\}^{-1} > 0$ should be satisfied, i.e. the following expressions must be satisfied:

$$B < \frac{\varepsilon_F}{\mu}\sqrt{\frac{2}{1 + \frac{3\alpha n}{2\varepsilon_F}}} = B_0 \tag{5}$$

or

$$\alpha < \frac{2\varepsilon_F}{3n}\left[2\left(\frac{\varepsilon_F}{\mu B}\right)^2 - 1\right] = \alpha' \tag{6}$$

When $T \gg T_F$,

$$1 + \frac{4}{3}\left(\frac{\mu B}{kT}\right)^2\left[1 + \frac{3\alpha n}{2kT}\right] > 0 \tag{7}$$

When $T > T_F$,

$$1 + \frac{4}{3}\left(\frac{\mu B}{kT}\right)^2\left[1 + \frac{3\alpha n}{2kT}\right] > \frac{n\lambda^3}{2^{9/2}}\left[1 + \frac{70}{3}\left(\frac{\mu B}{kT}\right)^2\right] \tag{8}$$

Under the condition of $T < T_F$, the stability condition of the system expression (5) also includes the case of $T \to 0$.

3. Results and discussion

3.1. The case of $T < T_F$

It can be known from (5) that, under the condition of low temperatures ($T < T_F$), if the system is stable, the external magnetic field should be less than B_0, which is the upper-limit value of the magnetic field to keep the system stable. If the repulsive interactions between particles go strong, the upper-limit magnetic field will become weak. Here, the magnetic field needed to keep the system stable will be still lower, and the range of magnetic fields fulfilling stability condition of the system will become narrower. If the attractive interactions between particles go strong, the upper-limit magnetic field will become strong, and the range of magnetic fields fulfilling stability condition of the system will become wide. This means that the condition of magnetic field can be easily achieved. From (5), we can obtain (6), which represents the required condition of interactions for the stability of the system when the magnetic field maintains a certain value. Obviously, $\alpha' > 0$ under the condition of a weak magnetic field. The expression (6) is valid naturally for the attractive interactions. Then, the system will be kept stable. However, as to the repulsive interactions, the condition of $\alpha < \alpha'$ must be satisfied in order to make the system stable. Thus, α' is the upper-limit value of repulsive interactions for the stability of the system.

3.2. The case of $T \gg T_F$

Under the condition of weak interactions, $|\alpha|n/\varepsilon_F < 1$, and so when $T \gg T_F$, $|\alpha|n/(2kT) \ll 1$. Namely, whether the interactions are attractive or repulsive, and no matter how strong the magnetic field is, the stability condition expression (7) can be satisfied, and the system can be kept stable.

3.3. The case of $T > T_F$

When $T > T_F$, the stability condition expression (8) can be expressed as follows:

$$\left(\frac{\mu B}{kT}\right)^2 > \frac{n\lambda^3/2^{9/2} - 1}{\frac{4}{3}[1 + \frac{3\alpha n}{2kT}] - \frac{70}{3}\frac{n\lambda^3}{2^{9/2}}} \tag{9}$$

The numerator of the right hand side of (9) is negative because $n\lambda^3 < 1$. If the denominator of it is positive, the stability condition expression (9) can be satisfied for any magnetic field. When the denominator of (9) is positive, the relation

$$\alpha > \frac{2kT}{3n}\left[\frac{70}{4}\frac{n\lambda^3}{2^{9/2}} - 1\right] \tag{10}$$

is required.

When $\alpha > 0$, i.e. the interparticle interactions are repulsive, the expression (10) can be satisfied naturally, and then, the stability condition of the system has no concrete requirement for magnetic field. When $\alpha < 0$, i.e. the interparticle interactions are attractive, the expression (10) can be expressed as

$$|\alpha| < \frac{2kT}{3n}\left[1 - \frac{70}{4}\frac{n\lambda^3}{2^{9/2}}\right] = |\alpha|_0 \tag{11}$$

That is to say, when the parameter of attractive interactions $|\alpha|$ is weaker than $|\alpha|_0$, the system will be always stable, no matter what value the magnetic field is.

If the denominator of the right hand side of expression (9) is negative, then it can be expressed as

$$B > \frac{kT}{\mu}\left|\frac{n\lambda^3/2^{9/2} - 1}{\frac{4}{3}\left[1 + \frac{3\alpha n}{2kT}\right] - \frac{70}{3}\frac{n\lambda^3}{2^{9/2}}}\right|^{1/2} = B_0 \tag{12}$$

where B_0 is the lower-limit magnetic field for the stability of the system, i.e. the value of magnetic field must be larger than B_0 in order to keep the system stable. If the denominator of the right hand side of expression (9) is negative, the following expression should be satisfied:

$$\alpha < \frac{2kT}{3n}\left[\frac{70}{4}\frac{n\lambda^3}{2^{9/2}} - 1\right] \tag{13}$$

It can be obviously found that, when $\alpha > 0$, the repulsive interactions make expression (13) invalid, and when $\alpha < 0$, the attractive interactions may make expression (13) valid, then expression (13) can be expressed as

$$|\alpha| > \frac{2kT}{3n}\left[1 - \frac{70}{4}\frac{n\lambda^3}{2^{9/2}}\right] = |\alpha|_0 \tag{14}$$

That is to say, under the condition $|\alpha| > |\alpha|_0$ of attractive interaction, a lower-limit magnetic field B_0 exists for the stability of the system. Here, the magnetic field must be stronger than the lower-limit magnetic field B_0 in order to keep the system stable. So, we can see that $|\alpha|_0$ is the dividing value of the attractive interaction, and the requirement of the stability for the magnetic field is determined by comparing the values of $|\alpha|$ and $|\alpha|_0$ of the attractive interactions. It can be also seen that this dividing value is related to the states (the temperature and the density of the particle number) of the system. Namely, the higher the temperature is, the larger the dividing value will be, and the bigger the density of particle number is, the smaller the dividing value will be.

3.4. The essence of the effects of interactions as well as magnetic field on the stability of the system

It can be known from statistic physics that if stability conditions cannot be satisfied, the homogeneous system will not stay at the stable equilibrium state, but will absolutely change its structure, i.e. it will change from the homogeneous system to the inhomogeneous one. The course of separation is phase transition. Therefore, the substance of the effects of magnetic field and interparticle interactions on the stability of the system are the effects of the two factors on the characteristics of the phase transition of it.

4. Conclusions

We have studied the thermal stability of a weakly interacting Fermi gas in a weak magnetic field in this study, and the exact analytical expressions of stability conditions at different temperatures have been proposed. We found that there is an upper-limit magnetic field for the stability of the system at the whole region of low temperatures ($T < T_F$, including $T \to 0$), i.e. the magnetic field must be weaker than the upper-limit magnetic field in order to keep the system stable. As for a certain weak magnetic field, if the interactions are repulsive, the condition of $\alpha < \alpha'$ must be satisfied for the stability of the system. At high temperatures ($T > T_F$), there is a dividing value $|\alpha|_0$ of attractive interactions. No matter how strong the magnetic field is, the system is stable for the case of repulsive interactions or for the case of the attractive parameter $|\alpha| < |\alpha|_0$. If the attractive parameter satisfies $|\alpha| > |\alpha|_0$, there is a lower-limit magnetic field for the stability of the system, i.e. the magnetic field must be stronger than the lower-limit magnetic field in order to keep the system stable. However, under the condition of very high temperatures ($T \gg T_F$), the system is always stable.

References

[1] Huang K, Yang CN. Quantum-mechanical many-body problem with hard-sphere interaction. Phys Rev 1957;105(3):767–75.
[2] Huang K, Yang CN, Luttinger JM. Imperfect Bose gas with hard-sphere interaction. Phys Rev 1957;105(3):776–84.
[3] Lee TD, Huang K, Yang CN. Eigenvalues and eigenfunctions of a Bose system of hard spheres and its low-temperature properties. Phys Rev 1957;106(6):1135–45.
[4] Lee TD, Yang CN. Low-temperature behavior of a dilute Bose system of hard spheres. Phys Rev 1958;112(5):1419–29.
[5] Tan WH, Yan KZ. A general approach to the Bose–Einstein condensation of neutral atoms with repellent interaction. Acta Phys Sin 1999;48(11):1983–91, [in Chinese].
[6] Shi HL, Zheng WM. Thermodynamic properties of a trapped interacting Bose gas. Phys A 1998;258(3):303–10.
[7] Wang C, Yan KZ. Numerical research on critical temperature of Bose–Einstein condensation for gas with interaction in harmonic trap. Acta Phys Sin 2004;53(5):1284–8, [in Chinese].

[8] Cui HT, Wang CL, Yi XX. Low-dimensional Bose–Einstein condensation in finite-number trapped atoms. Acta Phys Sin 2004;53(4):991–5, [in Chinese].
[9] Su GZ, Chen LX. Phase transition characteristics of Bose–Einstein condensation. Coll Phys 2003;22(8):8–10, [in Chinese].
[10] Yan KZ, Tan WH. Bose–Einstein condensation of neutral atoms with attractive interaction in a harmonic trap. Acta Phys Sin 2000;49(10):1909–11, [in Chinese].
[11] Baym G, Pethick CJ. Ground-state properties of magnetically trapped Bose-condensed Rubidium gas. Phys Rev Lett 1996;76(1):6–9.
[12] Dodd RJ, Edwards M, Williams CJ, et al. Role of attractive interactions on Bose–Einstein condensation. Phys Rev A 1996;54(1):661–4.
[13] Shuryak EV. Metastable Bose condensate made of atoms with attractive interaction. Phys Rev A 1996;54(4):3151–4.
[14] Modugno G, Roati G, Riboli F, et al. Collapse of a degenerate Fermi gas. Science 2002;297(5590):2240–3.
[15] Yuan DQ. The influence of weak interaction on thermodynamic properties and the stability of imperfect Bose gas. Acta Phys Sin 2006;55(4):1634–8, [in Chinese].
[16] Yuan DQ. The criterion of instability for a weakly interacting Fermi gas. Acta Phys Sin 2006;55(8):3912–5, [in Chinese].
[17] Men FD, Liu H. The instability conditions of a weakly interacting Fermi gas trapped in weak magnetic field. Chin Phys 2006;15(12):2856–60.
[18] Men FD. Thermodynamic properties of a weakly interacting Fermi gas in weak magnetic field. Acta Phys Sin 2006;55(4):1622–6, [in Chinese].