Alternative risk measure for decision-making under uncertainty in water management

Yueping Xu a, Yeou-Koung Tung b, Jia Li b, Shaofeng Niu a,*

a Department of Civil Engineering, Institute of Hydrology and Water Resources, Zhejiang University, Hangzhou 310028, China
b Department of Civil Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China

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Abstract

Taking into account uncertainties in water management remains a challenge due to social, economic and environmental changes. Often, uncertainty creates difficulty in ranking or comparing multiple water management options, possibly leading to a wrong decision. In this paper, an alternative risk measure is proposed to facilitate the ranking or comparison of water management options under uncertainty by using the concepts of conditional expected loss and partial mean. This measure has the advantages of being more intuitive, general and could relate to many other measures of risk in the literature. The application of the risk measure is demonstrated through a case study for the evaluation of flood mitigation projects. The results show that the new measure is applicable to a general decision-making process under uncertainty.

Keywords: Decision-making under uncertainty; Conditional expected loss; Partial mean; Water management; Risk measure

1. Introduction

Taking into account the ever-present uncertainty in water management remains a challenge for decision-makers and a hot research topic due to social, economic and environmental changes. Although the issues of decision-making under uncertainty have been addressed in the literatures [1,2], decision-makers are still reluctant to explicitly consider uncertainty in decision-making. This is due to the fact that the presence of uncertainties usually adds confusion and complexity to an already complicated task. One example of such confusion and complexity occurs in the evaluation or ranking of multiple viable management options. In water management, the ranking problem because of uncertainty has been investigated by several researchers [2–6]. Reda and Beck [4] and Duchnese et al. [5] utilized Monte Carlo simulations to generate model outputs for different management options under uncertainty and investigated the effects of uncertainty in model outputs on the ranking of storm water control management options.

As we know, in decision-making under uncertainty, risk is a frequently used criterion to evaluate the merit of management options. Conventionally, risk is defined as an exposure to undesirable consequences. According to Ref. [7], this risk has two components: (i) exposure and (ii) the nature of the undesirable consequences. Exposure is measured by the probability of encountering undesired consequences, while the second component depends on the nature of undesirable consequences and the metric used. In the literature, traditional risk measures which can be used to deal with the ranking problem include the expected value method, the min–max method and the Markowitz’s mean–variance analysis [8,9]. Though traditional, they still...
have a wide variety of applications in water engineering economic analysis. Duchene et al. [5] used the expected value method as the risk measure to rank storm water control management options. However, these risk measures are not general and have their limitations. In this paper, an alternative risk measure is proposed by incorporating the concepts of conditional expected loss and partial mean into the decision rule. Afterwards, its capability to rank management options under uncertainty is investigated in a flood mitigation project. This proposed risk measure is regarded as more intuitive, general and building blocks for computing other risk measures and applying such measure can add more reliability to decision-making under uncertainty.

2. Methods: proposed risk measure

As stated in several research works [6], due to uncertainty in performance variables, the ranking of different management options is not a trivial task. Illustrations of such ranking problem can be found elsewhere [2,6]. For the purpose of discussion, two management options, Option-1 and Option-2, are used herein to explain the decision rules.

Suppose that \( B_1 \) and \( B_2 \) are performance variables for these two management options, based on which the preferences are to be judged. Without the loss of generality, let \( B_1 > B_2 \) indicate that Option-1 is preferred over Option-2 (denoted by Option-1 \( \succ \) Option-2). When the performance variables \( B_1 \) and \( B_2 \) are subject to uncertainty, this decision situation under uncertainty can be cast into a statistical hypothesis test problem as shown in Table 1.

When Option-1 is better, the selection of Option-2 (or rejection of Option-1) would be an erroneous decision. By the same token, a wrong decision is committed by selecting Option-1 if Option-2 is truly a better one. When an erroneous decision is made in either case, loss will occur. On the other hand, there are two cases where correct decisions can be made, which correspondingly would be associated with some gains. In this paper, the loss or gain shown in Table 1 is the main decision criterion used to evaluate multiple water management options whose corresponding economic merits or utilities are subject to uncertainty.

2.1. Conditional risk measure

Herein, the concept of conditional expected loss is used because the losses and gains shown in Table 1 are all conditional. While this concept is often used in economic fields [10], it has not been used in water resource management. Therefore, it is meaningful to investigate its usefulness in water resources management. The new risk measure is derived by incorporating the concepts of conditional expected loss and partial mean.

The loss (gain) is the expected loss (gain) under the condition that one knows which management option is actually better, i.e. the state of nature. When Option-2 is actually better, the conditional expected loss associated with the selection of Option-1 can be expressed as

\[
E(\text{Loss due to selecting } O_1 \mid O_2 \text{ is better}) = E[L_1 \mid O_2]\\
= E[B_2 - B_1 \mid B_2 \geq B_1] = E[-(B_1 - B_2) \mid B_1 - B_2 \leq 0]
\]

where \( E[L_1 \mid O_2] \) represents the conditional expected loss due to selecting Option-1 when Option-2 is better (represented by \( B_2 \geq B_1 \)). Similarly, when Option-1 is better, the conditional expected loss associated with selecting Option-2, can be written as

\[
E[L_2 \mid O_1] = E[B_1 - B_2 \mid B_1 > B_2]\\
= E[B_1 - B_2 \mid B_1 - B_2 > 0]
\]

By the same token, the conditional expected gains corresponding to the selection of Option-1 and Option-2 can be calculated, respectively, as

\[
E[G_1 \mid O_1] = E[B_1 - B_2 \mid B_1 > B_2]\\
= E[B_1 - B_2 \mid B_1 - B_2 > 0] = E[L_2 \mid O_1]
\]

\[
E[G_2 \mid O_2] = E[B_2 - B_1 \mid B_2 \geq B_1] = E[L_1 \mid O_2]
\]

Eqs. (3) and (4) indicate that the conditional expected gain associated with one option is equal to the conditional expected loss for selecting the other option, which is rather intuitive. Therefore, one only needs to pay attention to the conditional expected losses. The conditional expected losses defined in Eqs. (1) and (2) are related to the conditional partial mean, which have been proposed by Buck and Askin [7]. The conditional partial mean has been used for economic risk that unifies many less general risk measures, such as the classical mean-variance analysis.

The partial mean of a random variable is its expected value over a specified range. If a random variable has a continuous probability density function \( f(x) \) over its entire range from \( x_{lb} \) to \( x_{ub} \), the partial mean over a partial range from \( c \) to \( d \) is defined as

\[
\mu_{(x_{lb} \rightarrow d)} = \int_{c}^{d} xf(x)dx \quad x_{lb} \leq c \leq x \leq d \leq x_{ub}
\]

The conditional partial mean is therefore defined as

\[
\mu_{d(x)} = \int_{c}^{d} xf(x)dx; \quad \mu_{u(x)} = \int_{x_{lb}}^{x_{ub}} xf(x)dx
\]
\[ \mu'_{(x-c|d)} = \frac{\mu_{(x-c|d)}}{\Pr(c \leq X \leq d)}; \quad \mu_{d(x|c)} = \frac{\mu_{d(x|c)}}{\Pr(X \geq c)} \]

where \( \Pr(X \leq c) \) is the non-exceedance probability. Applications of the partial mean can be found in economic fields [10,11]. In terms of conditional partial mean, Eqs. (1) and (2) can be expressed as

\[ E[L_1 | O_2] = \mu'_{u(b,0)} = \int_{-\infty}^{0} -bf_{B}(b)db / Pr(B \leq 0) \]

\[ E[L_2 | O_1] = \mu'_{u(b,0)} = \int_{0}^{\infty} bf_{B}(b)db / Pr(B > 0) \]

where \( B = B_1 - B_2; f_{B}(b) \) is the probability density function of a random variable \( B; \Pr(B \leq 0) \) and \( \Pr(B > 0) \) are the non-exceedance and exceedance probabilities, respectively.

Eqs. (8) and (9) are the conditional risk measure proposed in this paper. This risk measure can quantify the risk associated with the actions of a decision, assessing both the probability and magnitude of potential losses. Therefore, it is more general than the traditional risk measures, such as the expected value method, the min-max method and the Markowitz’s mean-variance analysis. In this sense, the conditional risk measure can be used as building blocks for computing other risk measures.

Table 2 shows the comparisons of the classical methods and the proposed new method. As presented in this table, the classical methods need less information than the new risk measure whereas they can only provide partial information on results. Furthermore, they are not general and concrete enough.

### 2.2. Decision rule

Without the loss of generality, it is assumed that a larger performance variable corresponds to a better management option. The decision rule for selecting between two management options can be established according to the associated conditional expected loss as

If \( E[L_1 | O_2] < E[L_2 | O_1] \), Option-1 is preferred (Option-1 > Option-2);

Otherwise, Option-2 is preferred (Option-2 > Option-1).

The ranking of multiple management options is mainly made by applying the above decision rule in a way of pairwise comparisons. This decision rule indicates that decision-makers prefer those management options associated with smaller expected losses when an erroneous decision is made, which is intuitive enough for understanding.

### 2.3. Calculation of risk measure

For maintaining reasonable complexity, it is assumed that both \( B_1 \) and \( B_2 \) are normally distributed. According to Ref. [12], the difference \( B \) between \( B_1 \) and \( B_2 \) is normally distributed. Suppose that the mean and standard deviation of \( B_i \) are \( \mu_{B_i} \) and \( \sigma_{B_i} \), respectively, for \( i = 1 \) and 2. The mean \( \mu \) and standard deviation \( \sigma \) of \( B_1 - B_2 \) are \( \mu_{B_1} - \mu_{B_2} \) and \( \sqrt{\sigma_{B_1}^2 + \sigma_{B_2}^2} \), respectively. Therefore, Eqs. (8) and (9) become

\[ E[L_1 | O_2] = \mu'_{u(b,0)} = \int_{-\infty}^{b} -bf_{B}(b)db / \Pr(B \leq 0) \]

\[ E[L_2 | O_1] = \mu'_{u(b,0)} = \int_{b}^{\infty} bf_{B}(b)db / \Pr(B > 0) \]

where \( \Phi \) represents the standard normal cumulative distribution.

Analytical formulations of non-exceedance probability and partial means for some simple distributions can be found in Ref. [11] and correspondingly the risk value for these distributions can be obtained. For other distributions, often numerical solutions are needed to calculate the conditional expected loss.

### 3. Results: NPV as performance variable

For demonstrating the use of the proposed risk measure, a case study is used herein. The case study is obtained from Xu [13] and aims to evaluate different flood mitigation options under uncertainty. The performance variable is the net present value (NPV) of flood damage [13,14].
NPV is defined as the sum of costs of different flood mitigation options and incurred benefits, e.g. sand and gravel extraction to the present value of flood damage. In this way, the larger the net present value becomes, the less desirable the option is. Correspondingly, the decision rule becomes: if \( E[L_i|O_j] < E[L_j|O_i] \), Option-2 is preferred (Option-2 \( > \) Option-1). The detailed model information can be found in Ref. [14].

In total, five flood mitigation options are considered, including the current situation (do-nothing case). These options are roughly described here: (i) Option-0: current situation; (ii) Option-1: broadening the main river channel by 25 m; (iii) Option-2: deepening the main river channel by 1 m; (iv) Option-3: embankments; and (v) Option-4: spatial planning options. In this case study, the uncertainties in the model inputs and parameters are considered since these two sources of uncertainty are widely recognized as important. If necessary, other uncertainties can be incorporated as well for further applications. The detailed uncertainty sources and their ranges can be found in Ref. [13]. In this study, uncertainties of different sources are assumed independent and are propagated into the NPV for each option by Latin Hypercube Simulation [15], which is more precise for generating random samples than Monte Carlo sampling, because the full range of the probability distribution is sampled more evenly. This sampling scheme is also more efficient since it needs far less simulation runs than normal Monte Carlo sampling for achieving same accuracy.

Kolmogorv–Smirnov goodness-of-fit test shows that the NPV from the five options are all normally distributed. For example, for Option-3, the test shows that the statistics value is 0.063 and the critical value is 0.089 at a significance level of 5%. Therefore, the null hypothesis cannot be rejected. Fig. 1 shows the probability density functions associated with the NPV for the five options under consideration. Table 3 shows the sample mean and standard deviation of the NPV value associated with different options. Both Fig. 1 and Table 3 show large uncertainty in the NPV, which implies the non-trivial nature of evaluating the five flood mitigation options.

Since the NPV associated with individual management option is normally distributed, the difference in NPVs between pairwise management options is normally distributed as well [12]. Based on Eqs. (10) and (11), Table 4 gives the calculated conditional risks associated with different pairwise management options.

On the basis of the aforementioned decision rule in Section 2, when \( E[L_i|O_j] < E[L_j|O_i] \), the first option (4th option) is the rejected one since, in this case study, a larger NPV leads to a less desirable option. Table 4 shows the rankings for pairwise management options as well. Therefore, the ranking based on the decision rule is Option-3 \( < \) Option-2 \( < \) Option-0 \( < \) Option-4 \( < \) Option-1. From this calculation, it is known that the information provided by the proposed risk measure is rather intuitive and can provide decision-makers with an easy understanding to make a confident decision.

Table 5 presents the rankings based on different methods. The expected value method and the new risk measure give the same ranking order, while the min–max method derives a different one. The min–max method is more suitable for pessimistic decision-makers. It is noted from this table that the Markowitz’s mean–variance rule does not lead to a conclusive ranking because several options with a higher mean performance value also have a higher standard deviation value. As pointed out by Xu and Tung [2], the rankings of management options are more or less robust to different ranking methods whereas knowing the risks involved in pairwise option comparisons can give

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**Table 3**

<table>
<thead>
<tr>
<th>Management options</th>
<th>NPV Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option-0</td>
<td>137.89</td>
<td>32.22</td>
</tr>
<tr>
<td>Option-1</td>
<td>153.98</td>
<td>17.62</td>
</tr>
<tr>
<td>Option-2</td>
<td>143.51</td>
<td>23.33</td>
</tr>
<tr>
<td>Option-3</td>
<td>150.53</td>
<td>20.96</td>
</tr>
<tr>
<td>Option-4</td>
<td>114.62</td>
<td>26.78</td>
</tr>
</tbody>
</table>

**Table 4**

<table>
<thead>
<tr>
<th>Pairwise options</th>
<th>Conditional risks</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option-0 and Option-1</td>
<td>12.59</td>
<td>Option-0 ( &lt; ) Option-1</td>
</tr>
<tr>
<td>Option-0 and Option-2</td>
<td>33.87</td>
<td>Option-0 ( &lt; ) Option-0</td>
</tr>
<tr>
<td>Option-0 and Option-3</td>
<td>35.74</td>
<td>Option-0 ( &lt; ) Option-3</td>
</tr>
<tr>
<td>Option-0 and Option-4</td>
<td>26.25</td>
<td>Option-0 ( &lt; ) Option-4</td>
</tr>
<tr>
<td>Option-1 and Option-2</td>
<td>89.64</td>
<td>Option-2 ( &lt; ) Option-2</td>
</tr>
<tr>
<td>Option-1 and Option-3</td>
<td>96.57</td>
<td>Option-3 ( &lt; ) Option-3</td>
</tr>
<tr>
<td>Option-1 and Option-4</td>
<td>62.85</td>
<td>Option-4 ( &lt; ) Option-4</td>
</tr>
<tr>
<td>Option-2 and Option-3</td>
<td>53.98</td>
<td>Option-0 ( &lt; ) Option-0</td>
</tr>
<tr>
<td>Option-2 and Option-4</td>
<td>20.05</td>
<td>Option-2 ( &lt; ) Option-2</td>
</tr>
<tr>
<td>Option-3 and Option-4</td>
<td>17.49</td>
<td>Option-4 ( &lt; ) Option-4</td>
</tr>
</tbody>
</table>
decision-makers more confidence to make a comfortable decision. The same ranking is rather reasonable since all the methods except the min–max method are largely based on the expected performance value calculated by the same sample generated through Latin Hypercube Simulation.

In this case study, goodness-of-fit test shows that the performance value is normally distributed. This certainly simplifies the calculation. If the performance value follows other distributions, numerical solutions are often needed. This only needs more computation and is not an obstacle for the application of the proposed risk criterion.

4. Discussion and conclusions

This paper presents and demonstrates the application of a new risk measure which is proposed based on the conditional expected loss and partial mean, to decision-making under uncertainty in water management. The traditional method of measuring risk is to define the risk in terms of the fluctuation of decision variable (e.g. investment return) around its expected value while the proposed risk measure encompasses both consequences and the degree of the undesirability. Compared to traditional methods, this risk measure has the advantages of easier interpretation, more general and could relate many of the previous measures of economic risk. In the meanwhile, the risk measure can provide more clear information about the actual differences of each individual management options and therefore decision-makers are better informed and expected to make a good decision for the situation under uncertainty. A side finding of this case study is that the ranking of different flood mitigation options is more or less robust to different methods, which is an interesting point for decision-makers.

Recommendations are given based on this paper. First, in order to take decision-makers’ opinion into account, it is advisable to consider the concept of acceptable risk. This value can be determined through interviews or questionnaire to relevant decision-makers. Therefore, the new decision rule can be modified as: if the acceptable risk > min \{ E[L_1 | O_2], E[L_2 | O_1] \}, information is sufficient for decision-makers to comfortably make a reasonable ranking under uncertainty. Second, the application of the new risk measure can be extended to multi-criteria decision-making which is highly relevant for water management problems due to the need of integrated assessment. The application of the new measure to the complicated multi-criteria decision-making is promising. However, it must be indicated that, during the multi-criteria decision analysis, the problem of preference cycle is often met. Solution to such problem can be found in Ref. [16].

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References