Three-dimensional investigation of velocity skin effect in U-shaped solid armature

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Abstract

A three-dimensional transient computational model is applied to investigate the velocity skin effect (VSE) in a U-shaped solid armature. With this model, current and heat transport in the armature can be evaluated accurately. The results show a local concentration of current and joule heating at the interface, and significant damage occurs at the edges of the armature. It is also observed that there is significant enhanced heating at sharp corners because of the armature geometry. The three-dimensional model can efficiently capture the salient features of the physical phenomena at the rail-armature interface, and contribute to the development of the armature design. © 2008 National Natural Science Foundation of China and Chinese Academy of Sciences. Published by Elsevier Limited and Science in China Press. All rights reserved.

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1. Introduction

In a typical railgun, the solid armature is considered as the carrier of current and Lorentz force. When the armature is accelerating, the current density near the surface of the conductor increases dramatically, which is often referred to as the velocity skin effect (VES). This effect could lead to the failure of solid armature, for example, the joule heating caused by VES at the rail-armature interface may erode and damage the armature [1].

The VES model captures the complex physical phenomena at the rail-armature interface, and has been intensively investigated by many researchers. Young and Hughes obtained some primary results on the current and magnetic field distributions in the armature with a steady velocity and found that the tendency of current concentration toward the rear of the armature is caused by the velocity skin effect [2]. Powell et al. analyzed the diffusion of electromagnetic field and the consequent nonuniform heating and extended the studies to various types of armatures, such as U-shaped, C-shaped, and the double-taper armature [3-8]. Later, three-dimensional codes, such as EMAP3D and DYNA3D, were developed by Hsich, Watt, et al. [9-12]. Most of these codes are based on the finite element method (FEM).

Based on the previous work, we have developed a three-dimensional time and position-dependent model of the rail and the U-shaped armature. We adopt the Douglas format of the finite difference method (FDM) and focus on the distributions of current and temperature at the interface which are strongly affected by velocity.

2. Calculation model

We set rails with enough length, initially, the armature is at the foreside of rails. Rails are made of copper, and the U-shaped armature is made of aluminum alloy. The x-axis is along the rail direction, and the z-axis is along the direction of the magnetic field between the rails with the y-axis
perpendicular to the \( x-z \) plane. Due to the symmetry of the problem, one can consider either of the two symmetric parts of the real model with the proper boundary conditions. The computational model has been divided into uniform cubic cells shown in Fig. 1.

3. Fundamental equations

The reduced magnetic-diffusion equation is given by

\[
\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} - \sigma \mu \frac{\partial B}{\partial t} = \sigma \frac{\partial B}{\partial t}
\]  

(1)

The energy-conservation equation is

\[
k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - c_v \frac{\partial T}{\partial t} = \rho c_v \left( \frac{\partial B}{\partial x} \right)^2 + \left( \frac{\partial B}{\partial y} \right)^2 + \left( \frac{\partial B}{\partial z} \right)^2
\]  

(2)

where \( B \) is the value of magnetic field in \( z \) direction, \( T \) is the temperature, \( \rho \), \( c_v \), \( \sigma \) and \( k \) represent the density, the specific heat, the electrical conductivity, and the thermal conductivity, respectively. \( \mu \) is the magnetic permeability of vacuum. During the projecting of the railgun, the armature feels a Lorentz force and accelerates along the positive direction of \( x \)-axis with the velocity denoted by \( v_x \). It is convenient to work in a frame of reference in which the armature is at rest, and the rail is moving with the velocity \(-v_x\).

4. Numerical method

Compared with FEM, which is generally used for that problem, FDM can be easily programmed with simpler format and smaller storage. Moreover, the convergence and stability of the FDM have been well studied. We choose Douglas scheme, which is given by [12]

\[
\frac{u^{k+1/2} - u^k}{\tau} = \frac{1}{h^2} \left[ \delta^2_x \left( \frac{u^{k+1/2} + u^k}{2} \right) + \delta^2_y \left( \frac{u^{k+1/2} + u^k}{2} \right) \right]
\]

(3)

At each step of this scheme, one needs to solve tri-diagonal matrix by using the chasing method.

5. Results and analyses

5.1. Initial and boundary conditions [6]

(1) The initial values of magnetic induction and temperature are chosen to be \( B_0 = 0 \), \( T_0 = 300 \) K, respectively.

(2) In the inner bore to the left of the armature, boundary conditions can be determined by Ampere’s law, i.e. \( B = \mu j \), where \( j \) is denoted as the total current per unit rail height, \( j = j_0 \sin(\pi t/t_p) \). The values of \( j_0 \) and \( t_p \) are 2.7 \times 10^4 A/m and 2.6 ms, respectively. It is assumed that the current enters parallel to \( x \)-axis from the left surface, and the end of the rail is far away from the initial position of the moving armature. So we have \( \frac{\partial B}{\partial x} = 0 \) on the left surface of the rail in the \( x \) direction. On the center surface of the armature, we have symmetry form as \( \frac{\partial B}{\partial y} = 0 \). On the other surfaces, \( B \) is set to zero.

(3) It is assumed that there is no heat transfer from the rail or armature to the atmosphere. On each conductor surface, we require \( \mathbf{n} \cdot \nabla T = 0 \).

(4) The rail and armature are composed of different materials, so that \( \frac{\partial B}{\partial y} \) and \( \frac{\partial B}{\partial z} \) are not continuous across the rail–armature interface. The appropriate conditions at the interface can be written as \( \frac{1}{\epsilon} \frac{\partial B}{\partial y} = 0 \) and \( k \frac{\partial B}{\partial z} = 0 \).

5.2. Results of calculation

Figs. 2 and 3 show the contours of magnetic induction in different planes at 0.3 ms. The larger values appear near the contact surface of the rail and the armature. The contours diffuse into the interior with intensity declining, and eventually concentrate toward the left corner at the rail–armature interface. Furthermore, we can also find in \( x-z \) planes that the contours are concentrated toward the two back edges of the armature symmetrically and diffuse taking the shape of a semicircle.

According to Maxwell’s equations, it can be proved that the current cannot cross the lines of constant magnetic induction [8]. So the current path in the \( z = 1.5 \) mm plane
at 0.3 ms can be determined by these lines shown in Figs. 4 and 5. The current vectors are shown in Figs. 6 and 7. In the case of \( v = 0 \), the current lines and vectors in the rail are roughly parallel to the \( x \)-axis. It diffuses equally into the rail and the diffusion depth is much the same along the direction \( x \). It also can be observed that the current passes through 50% of the distance across the interface. In the case of \( v \neq 0 \), as a result of velocity skin effect, the magnetic field cannot diffuse into the interior of the rail without enough time. Hence, the current is totally concentrated toward the left corner of the armature at the interface and then spreads throughout the entire armature from the corner region.

Figs. 8–11 show the distributions of current density \( J_x \), \( J_y \) along the outer contact edge of the armature and the rail at different times. It can be seen that the maximum of \( J_x, J_y \) both appear at the interface near the rear of the armature, and rapidly decrease to zero along the direction \( x \). Peak values of \( J_x, J_y \) get larger along with the increasing speed.
It can be seen that the maximum temperature appears in the two back corners of the armature near the interface resulting from the concentration of local heat in Figs. 12 and 13. But there is a slight distance between the location of the maximum and the interface as mentioned by Powell and Zielinski [6]. They attributed this to conductive cooling at the interface. The phenomenon occurs because the armature in motion constantly contacts with new, cold rail material. It should be observed that there is an enhanced heating at the sharp corners because of the armature geometry.

Fig. 14 shows the temperature along the outer contact edge of the armature at different times. Apparently, the largest temperature gradient appears at the back edge of the armature. The magnitude of the temperature drops rap-
idly along the direction \( x \). Furthermore, we can find the temperature gradient at the back edge increases rapidly with increasing velocity.

In Fig. 15, the maximum temperature in the armature is plotted as a function of time. As expected, the maximum temperature in the armature during acceleration rises linearly. Affected by the physical properties of the material, the temperature reaches the melting point of aluminum at 0.25 ms. This will lead to some complex physical phenomena at the rail-armature interface.

6. Conclusions

The three-dimensional transient model for VSE in a U-shaped armature has been presented in this paper. Numerical results have been obtained by using the Douglas difference scheme effectively.

Commutations show that the concentration of current occurs at the left-hand edge of the rail-armature interface during the motion of the armature. It will lead to high-local-temperature, melting and even phase transitions in the armature. Compared with zero-velocity calculation, VSE is proved to be responsible for the distributions of the current and the temperature. It is also observed that there is significant enhanced heating at sharp corners as a result of the type of armature geometry.

The model can capture the essential physical phenomena at the rail-armature interface efficiently and enables us to analyze the physical parameters qualitatively. We believe it would contribute to the development of the solid armature design and cast some light on the experiments of railguns.

References