BPF-based reconstruction algorithm for multiple rotation–translation scan mode

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Abstract

In industrial CT, it is often required to inspect large objects using short line-detectors. To acquire the complete CT data for the scanning slice of large objects using short line-detectors, some multi-scan modes have been developed. But the existing methods reconstructing an image from the data scanned by multi-scan modes have to rebin the data into fan-beam or parallel-beam form via data interpolation. The data rebinning process not only increases great computational cost, but also degrades image resolution. In this paper, we propose a backprojection-filtration (BPF)-based reconstruction algorithm for rotation–translation (RT) multi-scan mode. An important feature of the proposed algorithm is that data rebinning process is not introduced. The simulation results have verified the validity of the proposed algorithm.

Keywords: Backprojection-filtration; Reconstruction algorithm; Multiple rotation–translation scan mode; Fan-beam

1. Introduction

For industrial non-destructive testing (NDT) applications, it is often required to inspect large objects using short line-detector. Since the scanned slice of large objects cannot be completely covered within the field of view of the fan-beam formed by the source and the line-detector, several multi-scan modes have been developed to acquire the complete CT data. Of them, the rotation–translation–translation (RTT) multi-scan mode [1,2] and rotation–translation (RT) multi-scan mode [3,4] are typical for engineering. Since the flux output from X-ray source is not isotropic, RT and RTT multi-scan modes also reduce the differences of the flux intensities detected by different detector cells of the line-detector and produce images superior to those of long line-detector. In order to implement RTT scan, the turntable requires translation in two orthogonal directions. Compared with RTT scan mode, RT scan mode requires only one direction translation of the turntable, so, it is easier to carry out for engineering than RTT scan mode.

The existing methods to reconstruct CT image for multi-scan modes are all based on filtered-backprojection (FBP) algorithm [2–4]. In order to reconstruct CT image, they have to rebin the data scanned in multi-scan modes into parallel-beam or fan-beam form via data interpolation. As well known, the data rebinning process not only increases great computational cost, but also degrades image resolution. In this paper, we propose a novel reconstruction algorithm for the RT scan mode. An important feature of the proposed algorithm is that data rebinning process is not introduced. Therefore, it results in high computational efficiency and high spatial resolution of the reconstructed image. Another feature of the proposed algorithm is that it is suitable for acceleration by Graphic Processing Unit.

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Our reconstruction algorithm for RT multi-scan mode is based on backprojection-filtration (BPF) reconstruction algorithm. BPF algorithm was first proposed by Zou and Pan [5] to reconstruct exactly an image for helical cone-beam CT, which is another kind of exact reconstruction algorithm following after the kind of exact reconstruction in FBP format proposed originally by Katsevich [6]. An explicit BPF formula for 2D image reconstruction was proposed by Noo et al. [7] using the finite inversion formula of Hilbert transform. A review in detail on the recent development and historical relationship on FBP and BPF researches was given by Wang et al. [8]. The main idea of BPF is to obtain the so-called differentiated backprojection (DBP) image by backprojecting the derivative of the projections. Since the DBP image is related to the Hilbert image along some direction [7], the CT image can be recovered from the Hilbert image using the finite inverse formula of Hilbert transform. Being aware that the Hilbert image is independent of CT scan mode, we derive the DBP formula to obtain the Hilbert image for the RT multi-scan mode. The key point of the DBP formula is how to weigh each set of the derivatives of the RT multi-scan projections, and how to merge them forming an entire DBP image. Noting the localized feature of the derivative of the projections and coordinates relationship of each of the sets of the RT multi-scan projections, we derive the formula to obtain the entire DBP image from the sets of the derivative of the projections. For simplicity, the derivation for our formula is limited to the RT three-scan mode, which can be extended to the general RT multi-scan as well as the RTT multi-scan without difficulty.

2. RT multi-scan mode and its virtual equivalence

In this section, we introduce the RT multi-scan mode and its virtual equivalence. For sake of simplification, we only introduce the RT three-scan mode here.

As shown in Fig. 1, let \( S_0 \) denote the X-ray focus, the thick line denote the line-detector with equidistant collinear cells, and \( O_D \) denote the center of the line-detector. Suppose that \( O_D S_0 \) is vertical to the line-detector. The coordinates system \( Oxy \) is settled as follows: \( O \) is the origin located on the line segment \( O_D S_0 \), and the \( x \) axis and the \( y \) axis are vertical and parallel to \( O_D S_0 \), respectively, forming a right-hand system. Let \( R_O \) denote the distance between \( S_0 \) and \( O \), \( R_D \) denote the distance between \( S_0 \) and \( O_D \), and \( l \) denote the length of the line-detector. Three points, \( O_- \), \( O \) and \( O_+ \), lie on the \( x \) axis and satisfy \( |OO_+| = |OO_-| < R_O / R_D \).

In the RT three-scan mode the X-ray source and the line-detector are fixed, and the scanned object is placed on the turntable. The center of the turntable can be translated to the positions \( O_- \), \( O \) and \( O_+ \), respectively. The line-detector collects the flux of X-ray penetrating through the scanned object when the turntable is rotating in full turn around the centers \( O_- \), \( O \) or \( O_+ \). The three sets of projections corresponding, respectively, to \( O_- \), \( O \) and \( O_+ \) are obtained from the three rotations.

For the convenience of the derivation in Section 4, we introduce a virtual scan mode which is equivalent to the RT three-scan mode. It is obvious that the translation of a turntable along the \( x \) axis corresponding to a stationary fan-beam is equivalent to a translation of a fan-beam along the opposite direction of the \( x \) axis corresponding to a stationary turntable, and a rotation of a turntable around \( O \) clockwise is equivalent to a rotation of a fan-beam around a center \( O \) of a stationary turntable anti-clockwise. Therefore, RT three-scan mode in Fig. 1 is equivalent to the virtual scan mode in Fig. 2, where the X-ray focus \( S_0 \) of the middle fan-beam is located on the circle with the center \( O \) and the radius \( R_O \), while the X-ray focuses \( S_\pm \) of the two-side fan-beams are both located on the circle with the center \( O \) and the radius \( \sqrt{R_O^2 + |OO_\pm|^2} \). The equivalent scan mode can be regarded as a generalization of the fan-beam scan,
3. BPF algorithm for parallel-beam scan mode

As a foundation of the next section, we introduce the BPF reconstruction algorithm for the parallel-beam scan mode in this section. We begin with introducing the Hilbert transform on 1D and 2D function. As will be shown, the result of backprojecting the derivative of the projections will produce a Hilbert image. And then we describe how the original image can be recovered from a Hilbert image [7].

The Hilbert transform on 1D function (say \( g(s) \)) is defined by a convolution with the kernel \( 1/(\pi s) \),

\[
Hg(s) = \int_{-\infty}^{\infty} \frac{g(s')}{\pi(s-s')} ds'
\]

If there are \( L \) and \( U \) (satisfying \( U > L \)) so that \( Hg(s) \) is known inside the interval \([L,U]\) and if there is some small positive \( \epsilon \) so that \( g(s) = 0 \) for \( s \notin [L + \epsilon, U - \epsilon] \), then \( g(s) \) for all \( s \in [L + \epsilon, U - \epsilon] \) can be recovered by the finite inversion formula of the Hilbert transform given by Mikhlin [9],

\[
g(s) = -\frac{1}{\sqrt{(s-L)(U-s)}} \times \left( \int_{L}^{U} \sqrt{(s'-L)(U-s') \frac{Hg(s')}{\pi(s-s')}} ds' + C \right)
\]

where \( C \) is a constant which can be determined from knowledge of \( g(s) \) at some \( s \in [L,L + \epsilon] \cup [U - \epsilon, U] \) [7].

The Hilbert transform on 2D function \( f(x) \) along lines at angle \( \theta \) measured from the \( y \) axis anticlockwise is defined as

\[
H_0f(x) = \int_{-\infty}^{\infty} \frac{f((x \cdot \theta) + s \theta^\perp)}{\pi(x \cdot \theta - s)} ds
\]

where \( x = (x,y) \), \( \theta = (\cos \theta, \sin \theta) \) and \( \theta^\perp = (-\sin \theta, \cos \theta) \).

We now describe backprojection (DBP) of parallel-beam. Let \( f(x) \) be the density function of the scanned slice of object, where \( x = (x,y) \) denotes the coordinates of a pixel on the reconstructed image. Let \( p(\phi, r) \) denote the parallel-beam projections of \( f(x) \),

\[
p(\phi, r) = \int_{-\infty}^{\infty} f(r \phi + s \phi^\perp) ds
\]

where \( \phi = (\cos \phi, \sin \phi) \), \( \phi^\perp = (-\sin \phi, \cos \phi) \) is the angle of the normal of an X-ray measured from the \( x \) axis anti-clockwise, and \( r \) is the distance of the X-ray to the origin \( O \). The DBP formula \( b_d(x_0) \) of the parallel-beam is expressed in Ref. [7] as

\[
b_d(x_0) = \int_{0}^{\pi} \int_{-\infty}^{\infty} p(\phi, r) \text{sgn}(\sin(\phi - \theta)) \times \delta'(x_0 \cdot \phi - r) dr d\phi
\]

where \( \delta'(r) \) is the derivative of the Dirac function. It was proved in Ref. [7] that \( b_d(x_0) = -2\pi H_0f(x_0) \).

For fixed \( \theta \) and \( t \), if there are \( L_t \) and \( U_t \) (satisfying \( U_t > L_t \)) so that \( H_0f(x) \) is known for all \( x \cdot \theta^\perp \in [L_t,U_t] \), and if there is some small positive \( \epsilon_t \) so that \( f(x) = 0 \) for all \( x \cdot \theta^\perp \notin [L_t + \epsilon_t, U_t - \epsilon_t] \), then \( f(x_0) \) for any \( x_0 \) on line \( x \cdot \theta = t \) satisfying \( x \cdot \theta^\perp \in [L_t + \epsilon_t, U_t - \epsilon_t] \) can be recovered by the following finite Hilbert inverse formula

\[
f(x_0) = \frac{-1}{\sqrt{(x_0 \cdot \theta^\perp - L_t)(U_t - x_0 \cdot \theta^\perp)}} \times \left( \int_{L_t}^{U_t} \sqrt{(s-L_t)(U_t-s)} \times H_0f((x_0 \cdot \theta^\perp - s \theta^\perp) / \pi(x_0 \cdot \theta^\perp - s)) ds + C \right)
\]

where the constant \( C \) needs to be calculated for each \( t \), which can be determined from knowledge of \( H_0f(x) \) at some \( x \cdot \theta^\perp \in [L_t + \epsilon_t, U_t - \epsilon_t] \) on the line \( x \cdot \theta = t \) [7].

4. DBP formula for RT three-scan mode

As we mention above, the key point to the parallel BPF algorithm is to obtain the Hilbert image from parallel projections along some direction. Being aware that the Hilbert image of a CT image is independent of CT scan mode, we will derive a formula to obtain the Hilbert image for the RT three-scan mode in this section. The problem we face is how to weigh each set of the derivatives of the RT three-scan projections to obtain three DBP images and how to merge them into an entire DBP image that is related to the Hilbert image.

First we describe some denotations for the RT three-scan mode. For simplicity, we will derive the DBP formula for the RT three-scan mode from its virtual equivalent in Fig. 2. Let \( h \) denote the translation distance of the rotation center of the turntable, namely, \( |OO_1| = |OO_2| = h \). Let \( \beta \) denote the angle formed by \( OS_0 \) with the \( y \) axis anti-clockwise. We define three imaginary detector coordinates axes \( u_-, \beta u_-, \) and \( u_+ \) with the same direction \( \beta = (\cos \beta, \sin \beta) \), but with different origins \( O_-, O \) and \( O_+ \), respectively. Let \( p_-(\beta, u_-), p(\beta, u) \) and \( p_+(\beta, u_+) \) represent the three sets of fan-beam projections emitted from \( S_-, S_0 \) and \( S_+ \), respectively. Let us keep in mind that \( p_- (\beta, u_-), p_0 (\beta, u) \) and \( p_+ (\beta, u_+) \) are known for \( u_-, u \) and \( u_+ \in [-|O_0R_d|/(2R_d), |O_0R_d|/(2R_d)] \).

We start our derivation from the parallel DBP formula (2) and rewrite it for a \( 2\pi \) scan

\[
b_d(x_0) = \frac{1}{2} \int_{0}^{2\pi} \int_{-\infty}^{\infty} p(\phi, r) \text{sgn}(\sin(\phi - \theta)) \times \delta'(x_0 \cdot \phi - r) dr d\phi
\]

where \( x_0 \) represents any given point on DBP image. We can construct an auxiliary function as follows. \( k_1(r) \) is an infinitely differentiable function satisfying conditions:

\[(i) \quad k_1(r) = 1 \quad \text{for} \quad r \geq \frac{R_0b}{\sqrt{\sin^2(b/2)}} + \frac{b}{2},\]
Now we can rewrite formula (4) as

$$b_{0, \pm}(x_0) = \frac{1}{2} \int_0^{2\pi} \frac{R_O}{(R_O - x_0 \cdot \beta_+ \mp h)^2} \frac{d}{du_{\pm}} \left\{ \left( \frac{R_O}{R_O^2 + u_{\pm}^2} \right)^2 \times \frac{d}{du_{\pm}} \left\{ \left( \frac{R_O}{R_O^2 + u_{\pm}^2} \right)^2 \right\} \right\} |_{u_{\pm} = u_0}$$

where $\beta_+ = (-\sin \beta, \cos \beta)$, $u_0 = \frac{R_O x_0 \cdot \beta_+}{R_O^2 + x_0^2}$ and $u_{0, \pm} = \frac{R_O (x_0 \cdot \beta_+ \mp h)}{R_O^2 + x_0^2}$.

Here $u_0$ and $u_{0, \pm}$ are the projection positions on the imaginary detector axes of $x_0$ from the X-rays emitting from $S_0$ and $S_{\pm}$, respectively.

For the middle fan-beam with the vertex $S_0$, similar to the derivation of the fan-beam DBP [7], we can conduct formula (10) from formula (7). Without loss of generality, we only derive formula (11) for $b_{0, +}(x_0)$ from formula (8). Formula (11) for $b_{0, -}(x_0)$ can be similarly derived from formula (6) without any difficulty.

To obtain formula (11) for $b_{0, +}(x_0)$, we need to substitute $\beta(\beta, u_+)$ and $p_+(\beta, u_+)$ for $(\varphi, r)$ and $p(\varphi, r)$ in (8). From the geometric relationship in Fig. 2, the X-ray emitting from $S_+$ can be expressed as a parameter equation

$$x = OS_+ + \tau(u_+ + h)\beta - OS_+,$$  \(\tau > 0\)  (12)

where $x$ is a point on the ray and $OS_+ = R_O \beta_+ + h$. It follows from formula (12) that the angle between the normal vector of X-ray and the $x$ axis is

$$\varphi = \beta + \tan^{-1} \left( \frac{u_+}{R_O} \right)$$

Obviously $r^2 = \min(x \cdot x)$. Then we get minimum point $\tau_0$ of $x \cdot x$ by solving the equation $\frac{d(x \cdot x)}{d\tau} = 0$. Substituting $\tau_0$ into $\sqrt{x \cdot x}$, we obtain the distance of the X-ray to the origin $O$.

$$r = \frac{R_O (u_+ + h)}{\sqrt{R_O^2 + u_+^2}}$$

We can express for any $x_0$ as

$$x_0 = (x_0 \cdot \beta) \beta + (x_0 \cdot \beta_+) \beta_+$$

It follows that

$$x_0 \cdot \phi = (x_0 \cdot \beta) \cdot \phi + (x_0 \cdot \beta_+) \sin(\beta - \varphi)$$

$$= (x_0 \cdot \beta) \cos(\beta - \varphi) + (x_0 \cdot \beta_+) \sin(\beta - \varphi)$$

$$= (x_0 \cdot \beta) \frac{R_O}{\sqrt{R_O^2 + x_0^2}} + (x_0 \cdot \beta_+) \frac{u_+}{\sqrt{R_O^2 + x_0^2}}$$

We easily obtain the following expression from Fig. 2,

$$(u_{0, +} + h)x_0 \cdot \beta_+ = R_O(u_{0, +} - x_0 \cdot \beta)$$

Combining (14)-(16), we obtain
\[ x_0 \cdot \phi - r = \frac{R_O - x_0 \cdot \beta^{-}}{\sqrt{R_O^2 + u^2}} (u_{0, +} - u_+). \] (17)

From formulas (13) and (14), we obtain \( dr d\phi = |J| du_+ d\beta \),

where

\[ J = \left| \begin{array}{ccc} \frac{\delta x}{\delta \phi} & \frac{\delta y}{\delta \phi} \\ \frac{\delta x}{\delta \phi} & \frac{\delta y}{\delta \phi} \end{array} \right| = \frac{R_O (R_O^2 - hu_+)}{(R_O^2 + u^2)^{3/2}}. \]

Noting that \( \frac{R_O - x_0 \cdot \beta^-}{\sqrt{R_O^2 + u^2}} > 0 \) in formula (17) and \( \delta(r) \) is homogeneous of degree-2, and substituting (13), (14), (17) and \( dr d\phi = |J| du_+ d\beta \) into (8), then we obtain a simplified \( b_{0,+}(x_0) \) in formula (11).

In order to determine the left-hand sides of formula (10) and (11), we should choose a small positive \( \varepsilon \) so that \( p_0(\beta, u) \) and \( p_{\pm}(\beta, u_{\pm}) \) are known on the supports of

\[ 1 - k_i \left( \frac{R_O^2 - hu_+}{\sqrt{R_O^2 + u^2}} \right) - k_i \left( -\frac{R_O^2}{\sqrt{R_O^2 + u^2}} \right) \quad \text{and} \quad k_i \left( \pm \frac{R_O (u_+ u_{\pm})}{\sqrt{R_O^2 + u^2}} \right), \]

respectively. In fact, under the assumption of \( h < IRCD^2 \), it is not difficult to verify that there exists a \( \varepsilon > 0 \) meeting the requirement above.

Thus, we obtain each DBP image \( b_{0,0}(x_0) \) and \( b_{0,\pm}(x_0) \) from \( p_0(\beta, u) \) and \( p_{\pm}(\beta, u_{\pm}) \) by (10) and (11) respectively, and then obtain the entire DBP image \( b_0(x_0) \) by adding them together. Since the DBP image \( b_0(x_0) \) is related to the Hilbert image of \( f(x_0) \) by formula \( b_0(x_0) = -2\pi H_0 f(x_0) \), we recover \( f(x_0) \) by formula (3).

For numerical implementation, we adopt the projection-driven method to calculate \( b_{0,0}(x_0) \) and \( b_{0,\pm}(x_0) \) from \( p_0(\beta, u) \) and \( p_{\pm}(\beta, u_{\pm}) \) by (10) and (11), which also fit to implement by Graphics Processing Unit which can speed up the reconstruction process dramatically.

5. Numerical simulations and conclusion

In this section, we conduct numerical simulations to validate and evaluate the proposed algorithm for RT multi-scan mode in Section 4 compared to the BPF and the conventional FBP. Here we use a Shepp–Logan phantom for medical CT to evaluate the reconstruction algorithms since there is no uniform phantom for industrial CT. The Shepp–Logan phantom is shown in Fig. 3, where the long half axis of the largest ellipse is 335.48 mm, the minimum and maximum density values are 0.5 and 3.0, respectively.

In numerical simulations, the scan geometric parameters are as follows: the distance between the focus and the origin \( R_O = 1100.0 \) mm, the distance from the focus to the origin \( R_O = 1100.0 \) mm, the distance from the focus to the origin.
line-detector $R_D = 1500.0$ mm, the length of the line-detector $l = 357.7$ mm. The line-detector consists of 1022 detector cells and the size of each cell is 0.35 mm. Under this assumption, the radius of the field of view of the fan-beam formed by the X-ray focus and the line-detector is 130.23 mm. Obviously, the phantom cannot be completely covered within the field of view of the fan-beam. Using the RT three-scan mode in Fig. 1, three sets of projections of the phantom are acquired, as shown in Fig. 4(a), (b) and (c), respectively. Each set of them has 720 projections over 360 degrees. The translation distance $h$ is 255.0 mm. Dislocation can be observed among the three sets of projections, due to the positions of the fan-beam vertexes.

Fig. 6. Complete fan-beam projections of Shepp–Logan phantom.

Fig. 7. Reconstructed images of the Shepp–Logan phantom, by (a) the algorithm for RT three-scan mode, (c) the BPF and (e) the FBP, respectively. (b), (d) and (f) are the corresponding profiles along the 579th row of the images.
We use $\theta = 0$ Hilbert filtering and $\epsilon = 0.55$ mm in image reconstruction. Three DBP images are calculated from the three sets of projections by formulas (10) and (11), as shown in Fig. 5(a), (b) and (c), respectively. We add three DBP images together by formula (9), and obtain an entire DBP image shown in Fig. 5(d). The image with pixels $1024 \times 1024$ is recovered from the entire DBP image by formula (3), as shown in Fig. 7(a).

Fig. 6 is the sinogram of the fan-beam projections of Shepp–Logan phantom scanned by a line-detector which is three times as long as the detector used in RT three-scan mode. Fig. 7(c) and (e) are the images reconstructed by the BPF and FBP from complete fan-beam projections in Fig. 6, respectively. The ramp filter is applied with a Hamming window in the implementation of the FBP. In Fig. 7(b), (d) and (f), the image profiles along the 579th row of the images in Fig. 7(a), (c) and (e) are displayed with solid curves, respectively. The corresponding true profile in the original phantom is plotted with the dotted curve. The profiles in Fig. 7 show that the entire image of phantom can be reconstructed by the BPF and FBP algorithms, respectively. The reconstructed images from noiseless data are displayed with solid curves, respectively. The corresponding true profile in the original phantom is plotted with the dotted curve. The profiles in Fig. 7 show that the entire image of phantom can be reconstructed by the proposed algorithm for RT three-scan mode being as good as those reconstructed by the BPF and FBP from complete fan-beam projections in Fig. 6.

To further evaluate the reconstruction algorithms, we calculate three picture distance measures [10] for images reconstructed by the BPF and FBP algorithms, respectively. Let $t_{u,v}$ and $r_{u,v}$ denote the densities at the pixel $(u,v)$, the $i$th row and the $i$th column, of a phantom and a reconstructed image, respectively, and $\bar{t}$ denote the average of the density values in the phantom. Let the phantom and a reconstructed image consist of $N \times N$ pixels. Three picture distance measures are defined as the following, respectively:

$$
\begin{align*}
    d &= \left( \frac{1}{N^2} \sum_{u=1}^{N} \sum_{r=1}^{N} (t_{u,v} - r_{u,v})^2 / \sum_{u=1}^{N} \sum_{r=1}^{N} (t_{u,v} - \bar{t})^2 \right)^{1/2} \\
    r &= \sum_{u=1}^{N} \sum_{r=1}^{N} |t_{u,v} - r_{u,v}| / \sum_{u=1}^{N} \sum_{r=1}^{N} |t_{u,v}| \\
    e &= \max_{1 \leq j \leq N/2} |T_{ij} - R_{ij}|
\end{align*}
$$

where

$$
\begin{align*}
    T_{ij} &= \frac{1}{4} (t_{2i-1,2j-1} + t_{2i-1,2j+1} + t_{2i+1,2j-1} + t_{2i+1,2j+1}) \\
    R_{ij} &= \frac{1}{4} (r_{2i-1,2j-1} + r_{2i-1,2j+1} + r_{2i+1,2j-1} + r_{2i+1,2j+1})
\end{align*}
$$

Three picture distance measures for three images in Fig. 7 are shown in Table 1. It can be known from the table that the three picture distances between the phantom and the images reconstructed by the BPF and FBP algorithms for RT three-scan mode are slightly smaller than those between the phantom and the image reconstructed by the proposed algorithm for RT three-scan mode.

To examine the stability with respect to noise, we generate noisy data by adding Gaussian noise to the noiseless projections. The standard deviation of the Gaussian noise is 0.8% of the maximum value of the noiseless RT projections in Fig. 4 and fan-beam projections in Fig. 6, respectively. Fig. 8(a), (b) and (c) are the images reconstructed from noisy data by the proposed algorithm for RT multi-scan, the BPF and FBP, respectively. Three picture distance measures for three images in Fig. 8 are shown in Table 2. It can be known from Table 2 that there is not much difference between three images in Fig. 8(a), (b) and (c) for suppression of image noise.

![Fig. 8. Reconstructed images of the Shepp–Logan phantom from noisy data, by (a) the algorithm for RT three-scan mode, (b) the BPF and (c) the FBP.](image)
The distinct vibration can be observed near the edge of the reconstructed images in Fig. 7, which is caused by the insufficient samples of projection angles. The amplitude of vibration can be reduced when the samples of projection angles over 360 degrees are increased. In Fig. 9, the profile along the 579th row of the images reconstructed by the proposed algorithm from 1800 projections is shown, in which the amplitude of vibration is much less than that in Fig. 7.

The quality of the CT image reconstructed from real RT multi-scan data is expected to be superior to those of long line-detector by the BPF and the FBP. It is because of the act that for the real industrial CT system, the flux output from X-ray source is not isotropic, and then the data acquired in RT multi-scan mode using short line-detector are relative to more uniform intensity of the flux output than the data acquired with a long line-detector.

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