Robust stabilization of multilinear interval plants

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Received November 16, 2001; revised January 14, 2002

Abstract The robust $D$-stability of a class of multilinear interval polynomials is considered. Some sufficient conditions are given to judge the robust $D$-stability of the uncertain systems. In short, an uncertain system is robustly $D$-stable if some linear matrix inequalities are solvable. Moreover, our results are applied to the robust stabilization of multilinear interval plants. Taking advantage of the uncertainty structure information, these results are computationally tractable and effective in practice.

Keywords: multilinear interval polynomial, robust stabilization, Kharitonov vertex set, Kharitonov edge set, extreme point set, exposed edge set.

The robust stability of uncertain systems is an active research area in recent years\textsuperscript{[1–5]}. One of the highlights is the Kharitonov Theorem\textsuperscript{[1]}, which gives a sufficient and necessary condition for the robust stability of interval polynomials, i.e. a family of real polynomials with independent coefficient perturbations. Another key result is the Box Theorem, which gives a reduced dimensional test for the case of linear combinations of interval polynomials. In the Box Theorem, $m^4$ edges must be checked to determine the stability of the entire family of polynomials, with $m$ denoting the number of interval polynomials appearing in the uncertain polynomial family.

In this paper, a family of multilinear interval polynomials is considered. The characteristic polynomial of a cascade connection of some single-input single-output (SISO) systems with unity feedback is a typical example of such models. There is considerable interest in the stability of such families in the robust control literature\textsuperscript{[3,4,6,7]}. The model $P(s) = U(s) V(s) + X(s) Y(s)$ is considered in Ref. [6], where $U(s), V(s), X(s)$ and $Y(s)$ are interval polynomials. Barnish gave a sufficient and necessary condition for the robust stability of $P(s)$, which is a condition related to frequency $\omega$. In Ref. [7], the multilinear version of the Box Theorem is given, which is also a reduced dimensional test.

The regions considered here are the linear matrix inequality (LMI) regions introduced in Ref. [8].

The problem to be addressed can be roughly formulated as follows: given a family of multilinear interval polynomials $\mathcal{H}(s)$ and an LMI region $D$ in the complex plane, provide computationally tractable criteria for determining the $D$-stability of $\mathcal{H}(s)$. In this paper, we reduce this problem to LMI feasibility problem. Since LMIs can be solved numerically using efficient optimization algorithms\textsuperscript{[9,10]}, such as interior point techniques, the results in this paper provide practical analysis and synthesis tools for robustness research, such as robust stabilization.

1 Preliminaries

In this section, some notations and definitions are introduced.

Let $g(x_1, \ldots, x_m)$ be a multilinear function of $x = (x_1, \ldots, x_m)$, i.e. when all but one components of $x$ are fixed, $g(x_1, \ldots, x_m)$ is linear in the remaining variant. Let $P_1(s), \ldots, P_m(s)$ be interval polynomials with degrees of $r_1, \ldots, r_m$, respectively. Take

$$
\mathcal{P}(s) = \{ P(s) = p_{i0} + p_{i1}s + \cdots + p_{in}s^n, \quad p_{ij} \in \{ p_{ij}^L, p_{ij}^U \} \}
$$

$$
P_i^j(s) = p_{i0}^U + p_{i1}s + p_{i2}s^2 + p_{i3}s^3 + p_{i4}s^4 + \cdots,
$$

$$
P_i^j(s) = p_{i0}^L + p_{i1}s + p_{i2}s^2 + p_{i3}s^3 + p_{i4}s^4 + \cdots,
$$

$$
P_i^j(s) = p_{i0}^U + p_{i1}s + p_{i2}s^2 + p_{i3}s^3 + p_{i4}s^4 + \cdots,
$$

$$
P_i^j(s) = p_{i0}^L + p_{i1}s + p_{i2}s^2 + p_{i3}s^3 + p_{i4}s^4 + \cdots.
$$

(1)
Define the Kharitonov vertex set, the Kharitonov edge set, the extreme point set and the exposed edge set of $\mathcal{R}(s)$ as follows:

$$\mathcal{K}(s) = \{P_1^0(s), P_2^0(s), P_3^0(s), P_4^0(s)\},$$

$$\mathcal{E}(s) = \{\lambda P_1^0(s) + (1 - \lambda) P_2^0(s), \lambda P_3^0(s) + (1 - \lambda) P_4^0(s), \lambda P_1^0(s) + (1 - \lambda) P_3^0(s), \lambda P_2^0(s) + (1 - \lambda) P_4^0(s)\},$$

$$\mathcal{R}_V(s) = |P(s) = p_{i0} + p_{i1}s + \cdots + p_{ir}s^r, \quad p_{ij} \in [p_{ij}^L, p_{ij}^U], \quad j \neq k, \quad \mathcal{P}_E(s) = \bigcup_{k=0}^{1} P_k(s).$$

In this paper, we consider the following uncertain family,

$$\mathcal{G}(s) = \{|g(P_1(s), \ldots, P_m(s))|; \quad P_i(s) \in \mathcal{R}(s), i = 1, \ldots, m|.$$  

The cascade connections of some interval transfer functions with unity feedback, or feedback single-input multi-output (SIMO) or multi-input single-output (MISO) uncertain systems with interval components will have characteristic polynomials in the form of (7). The Kharitonov vertex set and the extreme point set of $\mathcal{G}(s)$ are

$$\mathcal{K}(s) = \{|g(P_1(s), \ldots, P_m(s))|; \quad P_i(s) \in \mathcal{R}(s), i = 1, \ldots, m|; \quad \mathcal{G}_V(s) = \{|g(P_1(s), \ldots, P_m(s))|; \quad P_i(s) \in \mathcal{R}_V(s), i = 1, \ldots, m|.$$  

$D \subset C$ is an open convex set in the form of

$$D = \left\{ s \in C; \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathcal{B}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} < 0 \right\},$$

which is also called an LMI region, where $\mathcal{B} = \mathcal{B}^*$ is a $2 \times 2$ matrix. When $\mathcal{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, the corresponding LMI region $D$ is the left half plane (LHP). In this case, $D$-stability is also called Hurwitz stability. Let

$$\mathcal{B} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

be a $2r \times (r + 1)$-dimensional matrix and $r$ is the degree of polynomial $g(P_1(s), \ldots, P_m(s))$.

The following lemma is due to Henron. Let $\mathcal{A}(s)$ denote the corresponding coefficient matrix of $\mathcal{A}(s)$.

**Lemma 1.** Suppose $D \subset C$ is a simply-connected region, $P(s)$ is an interval polynomial. Let $F_1(s), F_2(s)$ be two fixed polynomials. Then, the whole uncertain polynomial family $P(s) F_1(s) + F_2(s)$ is robustly $D$-stable if and only if

$$\|P(s) F_1(s) + F_2(s); P(s) \in \mathcal{P}_E(s)\|$$

is robustly $D$-stable.

Proof. By the Zero Exclusion Principle, $\|P(s) F_1(s) + F_2(s)\|$ is robustly $D$-stable if and only if:

(i) There exists one polynomial being $D$-stable in this family;

(ii) $0 \notin \{P(s) F_1(s) + F_2(s); \quad P_i(s) \in \mathcal{P}_E(s), \forall s \in \partial D\}.

The latter holds if $\|P(s) F_1(s) + F_2(s); P_i(s) \in \mathcal{P}_E(s)\|$ is robustly $D$-stable. Sufficiency is proved.

Necessity is obvious since $\|P_i(s) F_1(s) + F_2(s); P_i(s) \in \mathcal{P}_E(s)\|$ is robustly $D$-stable.

**Lemma 2.** (Box Theorem[4]) Given $m$ fixed real polynomials $F_1(s), \ldots, F_m(s)$ and $m$ interval polynomials $P_1(s), \ldots, P_m(s)$. The $\sum_{i=1}^{m} P_i(s) F_i(s)$ is robustly Hurwitz stable for all $P_i(s) \in P_i(s)$ if and only if $\sum_{i=1}^{m} P_i(s) F_i(s)$ is robustly Hurwitz stable for all $P_1(s) \times \cdots \times P_m(s) \in \bigcup_{i=1}^{m} P_{iV}(s) \times \cdots \times P_{mV}(s)$.

**2 Robustness stability**

In this section, we will consider the robustly stability of the uncertain family in (7).

**Theorem 1.** $\mathcal{G}(s)$ is robustly $D$-stable if there exist some matrices $\mathcal{P} = \mathcal{P}^*$, $\mathcal{P}$ solving the LMI feasibility problem
for all \( B_i(s) \in \mathcal{B}_i(s) \), where \( \mathcal{B}_i \) denotes the coefficient vector of \( B_i(s) \).

Proof. Construct an uncertain family as follows,

\[
\mathcal{B}_k(s) = \left\{ g(P_1(s), \ldots, P_m(s)) : \\
P_i(s) \in \mathcal{P}_i(s), k < i < m + 1 \\
P_i(s) \in \mathcal{P}_{1E}(s), 1 \leq i \leq k \right\}
\]

It is easy to see that

\( \mathcal{B}_m(s) \subset \mathcal{B}_{m-1}(s) \)
\( \subset \mathcal{B}_2(s) \subset \mathcal{B}_1(s) \subset \mathcal{B}(s) \).

We will establish the following relationship, i.e.

\( \mathcal{B}(s) \) is robustly D-stable
\( \iff \mathcal{B}_1(s) \) is robustly D-stable. \( (9) \)

When \( k = 1 \), \( (9) \) is the following one,

\( \mathcal{B}_1(s) \) is robustly D-stable
\( \iff \mathcal{B}(s) \) is robustly D-stable.

Necessity is obvious since \( \mathcal{B}_1(s) \subset \mathcal{B}(s) \). Assume \( \mathcal{B}_1(s) \) being robustly D-stable. Our aim is to prove that \( \mathcal{B}(s) \) is robustly D-stable. For all \( g(s) \) in \( \mathcal{B}(s) \), there exists \( P_i(s) \in \mathcal{P}_i(s) \) satisfying \( g(s) = g(P_1(s), \ldots, P_m(s)) \). Since \( g(s) \) is linear in \( P_1(s) \) when \( P_2(s), \ldots, P_m(s) \) are fixed, we have

\[
g(P_1(s), \ldots, P_m(s)) = P_1(s)g_1(P_2(s), \ldots, P_m(s)) + g_0(P_2(s), \ldots, P_m(s)).
\]

With those fixed \( P_2(s), \ldots, P_m(s) \), we define two uncertain families:

\[
H_1(s) = [P_1(s)g_1(P_2(s), \ldots, P_m(s)) + g_0(P_2(s), \ldots, P_m(s)) : P_1(s) \in \mathcal{P}_1(s)],
\]

\[
H_1(s) = [P_1(s)g_1(P_2(s), \ldots, P_m(s)) + g_0(P_2(s), \ldots, P_m(s)) : P_1(s) \in \mathcal{P}_{1E}(s)]
\]

By Lemma 2,

\( H(s) \) is robustly D-stable
\( \iff H_1(s) \) is robustly D-stable.

Obviously, \( g(P_1(s), \ldots, P_m(s)) \in H(s) \) and \( H_1(s) \subset \mathcal{B}_1(s) \). Therefore, \( g(P_1(s), \ldots, P_m(s)) \) is robustly D-stable. By definition, \( \mathcal{B}(s) \) is robustly D-stable.

If \( (9) \) holds for \( k = t \), we are to verify that \( (9) \) holds for \( k = t + 1 \) too. Necessity is obvious since \( \mathcal{B}_{t+1}(s) \subset \mathcal{B}(s) \). Assume \( \mathcal{B}_{t+1}(s) \) being robustly D-stable. For all \( g(s) \in \mathcal{B}_t(s) \), there exist \( P_i(s) \in \mathcal{P}_{1E}(s) \) for \( i = 1, \ldots, t \) and \( P_i(s) \in \mathcal{P}_i(s) \) for \( i = t + 1, \ldots, m \) satisfying

\[
g(s) = g(P_1(s), \ldots, P_m(s))
\]

Since \( g(P_1(s), \ldots, P_m(s)) \) is linear in \( P_{t+1}(s) \), there exist two polynomials

\[
g_2(s) = g_2(P_1(s), \ldots, P_{t+1}(s), \ldots, P_m(s)),
\]

\[
g_3(s) = g_3(P_1(s), \ldots, P_{t+1}(s), \ldots, P_m(s))
\]

such that

\[
g(P_1(s), \ldots, P_m(s)) = P_{t+1}(s)g_3(s) + g_2(s)
\]

Take

\[
H_2(s) = |P_{t+1}(s)g_3(s) + g_2(s), P_{t+1} \in \mathcal{P}_{t+1}(s)|,
\]

\[
H_3(s) = |P_{t+1}(s)g_3(s) + g_2(s), P_{t+1} \in \mathcal{P}_{t+1E}(s)|
\]

By Lemma 2,

\( H_2(s) \) is robustly D-stable
\( \iff H_3(s) \) is robustly D-stable.

Apparenty, \( H_2(s) \subset \mathcal{B}_t(s), H_3(s) \subset \mathcal{B}_{t+1}(s) \) and \( g(P_1(s), \ldots, P_m(s)) \in H_2(s) \). Thus, the condition that \( \mathcal{B}_{t+1}(s) \) is robustly D-stable implies that \( g(P_1(s), \ldots, P_m(s)) \) is robustly D-stable for all \( g(P_1(s), \ldots, P_m(s)) \in \mathcal{B}_t(s) \). That is to say, \( \mathcal{B}_t(s) \) is robustly D-stable. By assumption, \( (9) \) is true when \( k = t \). This condition implies that \( \mathcal{B}_{t+1}(s) \) is robustly D-stable. Hence, sufficiency is proved.

On the whole, we have verified that \( (9) \) is true when \( k = t + 1 \). By induction, \( (9) \) holds for every \( k \in [1, \ldots, m] \).

In particular, \( \mathcal{B}(s) \) is robustly D-stable if and only if \( \mathcal{B}_m(s) \) is so, where

\[
\mathcal{B}(s) = \{ g(P_1(s), \ldots, P_m(s)) : \\
P_i(s) \in \mathcal{P}_i(s), i = 1, \ldots, m \},
\]

\[
\mathcal{B}_m(s) = \{ g(P_1(s), \ldots, P_m(s)) : \\
P_i(s) \in \mathcal{P}_{1E}(s), i = 1, \ldots, m \}.
\]

Denote the convex hull of the set \( \mathcal{B}(s) \) as \( \text{conv} \mathcal{B}(s) \).

In the sequel, we will show that \( \mathcal{B}_m(s) \subset \text{conv} \mathcal{B}_t(s), \forall s \in C \).

For all \( P_i(s) \in \mathcal{P}_{1E}(s) \), there exist a real number \( \lambda_i \in [0, 1] \) and two polynomials \( P_i^{(1)}(s), P_i^{(2)}(s) \in \mathcal{B}_{1E}(s) \) satisfying

\[
P_i(s) = \lambda_i P_i^{(1)}(s) + (1 - \lambda_i) P_i^{(2)}(s)
\]

For all \( s \in C \), \( g(P_1(s), \ldots, P_m(s)) \in \mathcal{B}_m(s) \), since \( g(P_1(s), \ldots, P_m(s)) \) is a multilinear function of \( P_i(s) \),

\[
g(P_1(s), \ldots, P_m(s)) = \lambda_1 g(P_1^{(1)}(s), P_2(s), \ldots, P_m(s)) + (1 - \lambda_1) g(P_1^{(2)}(s), P_2(s), \ldots, P_m(s)) = \lambda_2 g(P_1^{(1)}(s), P_2^{(1)}(s), \ldots, P_m(s)) + (1 - \lambda_2) g(P_1^{(2)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]

\[
+ (1 - \lambda_1)(\lambda_2 g(P_1^{(1)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]

\[
+ (1 - \lambda_1)(1 - \lambda_2) g(P_1^{(2)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]

\[
+ (1 - \lambda_1)(1 - \lambda_2) g(P_1^{(2)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]

\[
+ (1 - \lambda_1)(1 - \lambda_2) g(P_1^{(2)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]

\[
+ (1 - \lambda_1)(1 - \lambda_2) g(P_1^{(2)}(s), P_2^{(2)}(s), \ldots, P_m(s))
\]
 Continuing this process, we have $g(P_1(s), \ldots, P_m(s)) \in \text{conv} \mathcal{V}(s)$. Consequently, $\mathcal{B}_m(s) \subset \text{conv} \mathcal{V}(s), \forall \mathcal{V} \in \mathcal{C}$. Hence, $\text{conv} \mathcal{V}(s)$ is robustly $D$-stable \hspace{1cm} $\Rightarrow \mathcal{B}_m(s)$ is robustly $D$-stable.

The set $\mathcal{V}(s)$ contains $\prod_{i=1}^{m} 2^r + 1$ distinct elements, i.e.

$$\text{conv} \mathcal{V}(s) = \text{conv} \{g_1(s), \ldots, g_l(s)\},$$

where $g_i(s) = g_i(P_1(s), \ldots, P_m(s)), P_j(s) \in \mathcal{B}_j(s), j = 1, \ldots, m$ and $l = \prod_{i=1}^{m} 2^r + 1$. For all $g_i(s) \in \mathcal{V}(s)$, denote its coefficient vector as $\mathcal{G}_i$.

For all $g(s) = (P_1(s), \ldots, P_m(s)) \in \mathcal{B}_m(s)$, there exist $l$ real numbers $\mu_i \in [0, 1], \sum_{i=1}^{l} \mu_i = 1$ and $l$ polynomials $g_1(s), \ldots, g_l(s) \in \mathcal{V}(s)$ such that

$$g(P_1(s), \ldots, P_m(s)) = \sum_{i=1}^{l} \mu_i \mathcal{G}_i(s).$$

For every $i \in \{1, \ldots, l\}$,

$$\begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} \begin{bmatrix} \mathcal{A} \otimes \mathcal{B}_i \\ \mathcal{D} \end{bmatrix} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} < 0$$

$\Leftrightarrow$$\mathcal{A}^* \mathcal{B} \otimes \mathcal{B}_i + \mathcal{A}^* \mathcal{D} + \mathcal{B} \mathcal{B}_i < 0$

$\Rightarrow$$\sum_{i=1}^{l} \mu_i (\mathcal{A}^* \mathcal{B} \otimes \mathcal{B}_i + \mathcal{A}^* \mathcal{D} + \mathcal{B} \mathcal{B}_i) < 0$

$\Leftrightarrow$$\mathcal{A}^* \mathcal{B} \otimes \left( \sum_{i=1}^{l} \mu_i \mathcal{B}_i \right) \mathcal{B} + \left( \sum_{i=1}^{l} \mu_i \mathcal{G}_i \right) \mathcal{D} < 0.$

(10)

Let $\mathcal{K}$ be the right null space of $g(s)$. Multiplying (10) by $\mathcal{K}$ on the right and by $\mathcal{K}^*$ on the left, the inequality becomes

$$\mathcal{K}^* \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} \begin{bmatrix} \mathcal{A} \otimes \left( \sum_{i=1}^{l} \mu_i \mathcal{B}_i \right) \\ \mathcal{D} \end{bmatrix} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} \mathcal{K} < 0.$$

Since $\mathcal{K} = \sum_{i=1}^{l} \mu_i \mathcal{G}_i$, we have

$$\mathcal{K}^* \mathcal{A} = \left( \sum_{i=1}^{l} \mu_i \mathcal{G}_i \right) \mathcal{K} < 0.$$

From $\mathcal{K}^* \mathcal{K} > 0$, we have that $\sum_{i=1}^{l} \mu_i \mathcal{B}_i = 0$. By Lemma 1, our proof is completed.

Let $l_1 = 4^m$. In the special case of $D$ being LHP, by Lemma 3, following the similar line in the proof of Theorem 1, we get the following result.

**Theorem 2**. $\mathcal{B}(s)$ is robustly Hurwitz stable if there exist some matrices $\mathcal{P}_i = \mathcal{P}_i^*$, $\mathcal{Q}$ solving the LMI feasibility problem

$$\begin{bmatrix} \mathcal{R} \\ \mathcal{B} \end{bmatrix} \begin{bmatrix} \mathcal{B} \otimes \mathcal{P}_i \\ \mathcal{D} \end{bmatrix} \begin{bmatrix} \mathcal{R} \\ \mathcal{B} \end{bmatrix} < 0,$$

where $\mathcal{R} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is a $2 \times 2$ scalar matrix, and $\mathcal{B}_i$ is the coefficient vector of $\mathcal{B}_i(s) \in \mathcal{K}(s)$.

**Remark 1**. Theorem 2 is more effective than Theorem 1 when $D$ is the LHP. The number of LMIs is reduced to $4^m$ from $\prod_{i=1}^{m} 2^r + 1$. This is due to the advantage of Lemma 3.

### 3 Robust stabilization of multilinear interval plants

Let $\mathcal{N}(s), \mathcal{P}_i(s), \mathcal{U}(s)$ and $\mathcal{V}(s)$ be interval polynomials. Consider the unity feedback system with an uncertain controller connected to a cascade of several interval plants, as shown in the configuration below, where $\mathcal{P}_i(s) = \frac{\mathcal{N}_i(s)}{\mathcal{D}_i(s)}$ is a proper interval plant and $\mathcal{U}(s) = \frac{\mathcal{V}(s)}{\mathcal{V}(s)}$ is a proper interval controller. The characteristic polynomial family of the closed loop systems is

$$\mathcal{F}(s) = |\mathcal{U}(s) \mathcal{N}_1(s) \cdots \mathcal{N}_m(s) + \mathcal{V}(s) \mathcal{D}_1(s) \cdots \mathcal{D}_m(s)|.$$  (11)

Take

$$\mathcal{F}^K(s) = |\mathcal{U}^K(s) \mathcal{N}^K_1(s) \cdots \mathcal{N}^K_m(s) + \mathcal{V}^K(s) \mathcal{D}^K_1(s) \cdots \mathcal{D}^K_m(s)|.$$

**Theorem 3**. An interval controller $\mathcal{U}(s)$ robustly stabilizes the uncertain systems shown in Fig. 1 if there exist some matrices $\mathcal{P}_i = \mathcal{P}_i^* > 0$, $\mathcal{Q}$ solving the LMI feasibility problem

$$\begin{bmatrix} \mathcal{R} \\ \mathcal{B} \end{bmatrix} \begin{bmatrix} \mathcal{B} \otimes \mathcal{P}_i \\ \mathcal{D} \end{bmatrix} \begin{bmatrix} \mathcal{R} \\ \mathcal{B} \end{bmatrix} < 0, \quad i = 1, \ldots, 4^{2m+2};$$
where $\mathcal{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathcal{F}$ denotes the coefficient vector of $f_i(s) \in \mathcal{F}_i(s)$.

**Fig. 1.** A unity feedback interconnected system.

**Remark 2.** For a control system containing several interconnected subsystems with uncertain parameters, as considered in this section, Theorem 3 provides a useful tool for solving the stability of the whole closed loop systems and Theorem 2 can be used to solve robust perform system with a fixed controller and an interval plant is a special case of (11).

**Remark 3.** By the Box Theorem[4] and its multilinear version, there exists a reduced dimensional test of the robust Hurwitz stability of system (11). In general, the vertex verification does not hold. A direct method is based on the Zero Exclusion Principle. For any fixed $\omega \in R$, evaluate the whole family at an $s = j\omega$, grid the uncertain set, plot the image set in the complex plane, check that the origin is excluded from the image set, and repeat the whole procedure for every $\omega \in R$. Such a method is computational tractable and not feasible, generally speaking. Theorem 3 overcomes these difficulties. Based on LMI, Theorem 3 gives an effective approach to checking the robust Hurwitz stability of control systems shown in Fig. 1.

**Remark 4.** Theorems 2 and 3 can be applied to MISO or SIMO systems. In that case, the characteristic polynomial of control system has exactly the form of (11).

In this paper, we provide a numerically efficient criterion for robust stability of multilinear interval polynomials based on LMI. This result is applied to robust stabilization of multilinear interval plants.

**References**


