Determining of cloud-top height from stereoscopic observation

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Abstract A new and accurate method is presented based on the cloud movement (height and position), the spherical and plane triangular relationships of the spacecraft, the center of the earth, the projected-cloud and the true-cloud for determination of cloud-top height and position. Synthetic stereo images that have spatial resolution of 1.25 km from a single satellite are used to test this method. It is demonstrated that the cloud-top structure can be determined from the stereoscopic measurements of geo-synchronous satellite with vertical accuracy of approximately 500 m. The vertical accuracy can be better with lower orbiters.

Keywords: stereoscopic observation, cloud-top height, spherical triangle.

Information on cloud-top height is very important in severe storm research, especially for cloud structure and dynamic analysis. Infrared cloud-top height is dependent on knowledge of cloud emissivity, ambient temperature, and the environmental lapse rate; it also requires the assumption that the cloud is in local thermodynamic equilibrium. In many important cases, cloud emissivity and the atmospheric temperature height profile are unknown and the cloud is not in equilibrium with its environment. This can lead to errors of 1~2 km or greater in infrared cloud-top height[1]. For an ideal stereo observation, two satellites simultaneously scan the same geographical area from different view angles. The resulting parallax between the two images of an observed feature is proportional to its height. Stereo measurements depend only on the basic geometric relationships of the observed visible cloud particles, and the cloud-top height is independent of the physical presentation of cloud.

Fujita presented an approximate but accurate method of stereo-height computations from overlapping images of geo-synchronous satellites[2]. GOEs-E, GOEs-W and GMS geo-synchronous satellites were used to determine the cloud-top height by Hasler and Fujita et al.[1~4]. The combination of GOEs with the NOAA low earth orbiters was also used in the computation of cloud-top height by Hasler[4]. It has been demonstrated that the cloud-top height can be measured with the height accuracy of about 500 m. These results show that the stereo cloud height measurements from geo-synchronous satellites can be achieved with high precision and the results can be more confirmative than heights estimated by infrared-based techniques.

Many researches have been done in the determination of stereo cloud-top height by Fujita and Hasler[1~4]. The basic expression by parallax in stereo height computations is

\[ H = \frac{P}{P_{10}} \times 10(\text{km}) \]

where \( P \) is the parallax of pixel in reference and test images; \( H \) the cloud-top height; \( P_{10} \) the parallax at the same position with 10 km height. For the computations of \( P \), Hasler used matches between a neighborhood surrounding reference image pixel and neighborhoods within a search area in the test image. Reference or test images must be remapped into the coordinate system of the other so that the feature matching can be done with minimum distortion. Eq. (1) is correct only when two satellites and the true-cloud are collinear; otherwise Eq. (1) is only an approximate method of cloud-top height computations.

In this paper, a new and accurate method is presented for determination of cloud-top height and position based on cloud movement (height and position), the spherical and plane triangular relationships of the satellite, the center of the earth, the projected-cloud and the true-cloud. Several stereo image pairs simulated for GMS-5 and FY-2 have been used to demonstrate the performance that can be expected in real data.
1 Methods

1.1 The geometrical relationship between projected-cloud and true-cloud

The geometric relation between the projected-cloud (mapped to the earth) and the true-cloud is presented in Fig. 1.

The mathematic relationships of the cloud-top height, the projected-cloud position and the true-cloud position will be presented as follows.

\[ \begin{align*}
\text{(i) Computations of true-cloud position } (\theta_T, \phi_T) \\
\text{by cloud-top height } (h) \text{ and projected-cloud position } (\theta_C, \phi_C). \\
\text{Computations of arc } CP = a \text{ in spherical triangle } P-N-C: \\
CN &= 90 - \phi_C, \quad PN = 90 - \phi_P, \\
\angle CNP &= \Delta \theta_{PC} = \theta_P - \theta_C, \\
\cos a &= \sin \phi_C \sin \phi_P + \cos \phi_C \cos \phi_P \cos(\Delta \theta_{PC}). \quad (2) \\
\text{Computations of } \angle CSO = \beta \text{ in plane triangle } S-C-O: \\
\overline{CO} &= r, \quad \overline{SO} = d, \\
\overline{SC} &= a_1 = \sqrt{r^2 + d^2 - 2rd \cos a}, \\
\cos \beta &= \frac{d - r \cos a}{a_1}. \quad (3) \\
\text{Computations of } \angle STO = \delta_1 \text{ in plane triangle } S-T-O: \\
\overline{TO} &= h + r, \quad \overline{SO} = d, \\
\sin \delta_1 &= \frac{d \sin \beta}{h + r}. \quad (4) \\
\text{Computations of } \angle SOT = a_1 \text{ and } \angle TOC = a_2 \\
\text{from } \beta \text{ and } \delta_1: \\
a_1 &= 180 - (\beta + \delta_1), \\
a_2 &= a - a_1. \quad (5) \\
\text{Computations of } \angle PCN = \gamma_1 \text{ in spherical triangle } N-P-C: \\
PN &= 90 - \phi_P, \quad CP = a, \\
\angle CNP &= \Delta \theta_{PC} = \theta_P - \theta_C, \\
\sin \gamma_1 &= \sin(90 - \phi_P) \sin(\theta_P - \theta_C) \sin(a). \quad (6) \\
\text{Computations of } \angle CNB = \mu_1, \angle CBN = \nu_1 \text{ and arc } BN = W_1 \text{ in spherical triangle } N-C-B: \\
U_1 &= CB = a_2, \quad V_1 = CN = 90 - \phi_C, \\
\angle PCN &= \gamma_1, \\
\mu_1 &= \arctan(m_1) + \arctan(m_2), \\
\nu_1 &= \arctan(m_1) - \arctan(m_2), \quad (7) \\
W_1 &= 2 \arctan \left( \frac{\cos \left( \frac{\mu_1 + \nu_1}{2} \right)}{\cos \left( \frac{\mu_1 - \nu_1}{2} \right)} \right) \tan \left( \frac{U_1 + V_1}{2} \right), \quad (8)
\end{align*} \]

where,
\[
m_1 = \frac{\cos \left( \frac{U_1 - V_1}{2} \right)}{\cos \left( \frac{U_1 + V_1}{2} \right)} \cot \left( \frac{\eta_1}{2} \right),
\]
\[
m_2 = \frac{\sin \left( \frac{U_1 - V_1}{2} \right)}{\sin \left( \frac{U_1 + V_1}{2} \right)} \cot \left( \frac{\eta_1}{2} \right).
\]

Computations of true-cloud position \((\theta_T, \phi_T)\):
\[\theta_T = \theta_C + \mu_1, \quad \phi_T = 90 - W_1. \quad (8)\]

(ii) Computations of projected-cloud position \((\theta_C, \phi_C)\) by cloud-top height \((h)\) and true-cloud position \((\theta_T, \phi_T)\).

Computations of arc \(BP = a_1\) in spherical triangle \(P-B-N\):
\[
BN = 90 - \phi_T, \quad PN = 90 - \phi_P, \quad \angle BNP = \Delta \theta_{PT} = \theta_P - \theta_T, \quad \cos \alpha_1 = \sin \phi_T \sin \phi_P + \cos \phi_T \cos \phi_P \cos (\Delta \theta_{PT}). \quad (9)
\]

Computations of \(\angle TSO = \beta\) in plane triangle \(S-T-O\):
\[
\overline{TO} = h + r, \quad \overline{ST} = a_2 = \sqrt{(r + h)^2 + d^2 - 2(r + h)d \cos \alpha_1}, \quad \cos \beta = \frac{d - (r + h) \cos \alpha_1}{a_2}. \quad (10)
\]

Computations of \(\angle SCO = \beta_2\) in plane triangle \(S-C-O\):
\[
CO = r, \quad SO = d, \quad \sin \beta_2 = \frac{d \sin \beta}{r}. \quad (11)
\]

Computations of \(\angle SOC = a\) and \(\angle TOC = a_2\) from \(\beta\) and \(\beta_2\):
\[
a = 180 - (\beta + \beta_2), \quad a_2 = a - a_1. \quad (12)
\]

Computations of \(\angle BPN = \eta_2\) in spherical triangle \(N-P-B\):
\[
BN = 90 - \phi_T, \quad BP = a_1, \quad \angle BNP = \Delta \theta_{PT} = \theta_P - \theta_T, \quad \sin \eta_2 = \frac{\sin(90 - \phi_T) \sin(\theta_P - \theta_T)}{\sin(a_1)}. \quad (13)
\]

Computations of \(\angle CNP = \mu_2\), \(\angle PCN = \nu_2\) and are \(CN = W_2\) in spherical triangle \(N-P-C\):
\[
U_2 = CP = a, \quad V_2 = PN = 90 - \phi_P, \quad \angle CPN = \eta_2.
\]
\[
\mu_2 = \arctan(m_3) + \arctan(m_4), \quad \nu_2 = \arctan(m_3) - \arctan(m_4), \quad (14)
\]
\[
W_2 = 2 \arctan \left[ \frac{\cos \left( \frac{\mu_2 + \nu_2}{2} \right)}{\cos \left( \frac{\mu_2 - \nu_2}{2} \right)} \right] \tan \left( \frac{U_2 + V_2}{2} \right),
\]
where,
\[
m_3 = \frac{\cos \left( \frac{U_2 - V_2}{2} \right)}{\cos \left( \frac{U_2 + V_2}{2} \right)} \cot \left( \frac{\eta_2}{2} \right),
\]
\[
m_4 = \frac{\sin \left( \frac{U_2 - V_2}{2} \right)}{\sin \left( \frac{U_2 + V_2}{2} \right)} \cot \left( \frac{\eta_2}{2} \right).
\]

Computations of projected-cloud position \((\theta_C, \phi_C)\):
\[\theta_C = \theta_p - \mu_2, \quad \phi_C = 90 - W_2. \quad (15)\]

1.2 Height computations with sub-image matches controlled by the cloud movement

Eqs. (2) ~ (15) permit us to compute the projected-cloud and the true-cloud position when the cloud-top height is given. Now, the key problem is how to get the cloud-top height.

Although the cloud-top height is unknown, we know that satellite, true-cloud and projected-cloud are collinear and that the cloud-top height is in a fixed range \([h_{\text{min}}, h_{\text{max}}]\) (e.g. \(h_{\text{min}} = 2.0 \text{ km}, \ h_{\text{max}} = 20.0 \text{ km}\)). Consequently, we can move the suppositional true-cloud in line to make a sequence of height level \([h_1, h_2, \ldots, h_i, \ldots, h_n]\) through a height interval; \(\Delta h\) (e.g. 0.1 km), where
\[
k = (h_{\text{max}} - h_{\text{min}})/\Delta h + 1
\]
and
\[
h_i = h_{\text{min}} + (i - 1)\Delta h.
\]
For a certain height level, any source cloud point in satellite-W (west) can be matched to the target cloud point in satellite-E (east) with Eqs. (2) ~ (15). A match score for source and target cloud points can be calculated. So, a sequence of target cloud points in satellite-E can be found for a fixed source cloud point in satellite-W with the true-cloud movement (both height and position are changed), and a sequence of match scores can be calculated with the different target points. The height with maximum match score is the cloud-top height.

The selection of height interval \((\Delta h)\) will affect
the accuracy of cloud-top height directly. The shorter the height interval is selected, the more calculation will be needed, and the better results will be obtained at last. Usually, the selection of height interval depends on the horizontal resolution in satellite image. For example, we can use $\Delta h = 0.25$ km when horizontal resolution is 1.0 km.

For any height level (height is $h_i (i = 1, 2, \cdots, k)$), the match procedures from the source cloud point to the target cloud point are as follows.

(i) Get the true-cloud position $(\theta_T, \phi_T)$ from the geocentric distance $(d_W)$, the terrestrial subpoints $(\theta_{W}, \phi_{W})$ and the source projected-cloud position $(\theta_{C-W}, \phi_{C-W})$ of satellite-W with Eqs. (2) − (8).

(ii) Get the target cloud position $(\theta_{C-E}, \phi_{C-E})$ in satellite-E from the geocentric distance $(d_E)$, the terrestrial subpoints $(\theta_{E}, \phi_{E})$ of satellite-E and the true-cloud position $(\theta_T, \phi_T)$ with Eqs. (9) − (15).

For the calculations of match scores of the source and target cloud points, a match is performed between a neighborhood surrounding source point (the "source sub-image") and neighborhoods surrounding target point (the "target sub-image"). Match scores are used to present the comparability of two sub-images. Statistical features (correlation coefficient, difference square sum, mean difference etc.) and structure features (texture, shape etc.) can be used as match scores. We combined the correlation coefficient (COEF) and the difference square sum (DSS) to make the match score in our test; the concave point in DSS with the bigger COEF is the best match; its match score is maximal, and its height is the cloud-top height. Fig. 2 shows the basic selection.

![Diagram](image_url)

**Fig. 2.** Height computations with sub-image matches controlled by the cloud movement. (a) The suppositional true-cloud movement in line (S_W to C_E) within the fixed height range [$h_{min}, h_{max}$]; (b) the match scores by matching between the source and target sub-images.

The following equations are for obtaining COEF and DSS with sub-image matrices $u_{m \times n}$ and $v_{m \times n}$ (each sub-image has $m$ rows and $n$ columns):

$$\text{COEF} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (u_{i,j} - \bar{u})(v_{i,j} - \bar{v})}{\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} (u_{i,j} - \bar{u})^2 \times \sum_{i=1}^{m} \sum_{j=1}^{n} (v_{i,j} - \bar{v})^2}}.$$  

(16)

where,

$$\bar{u} = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} u_{i,j}, \quad \bar{v} = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} v_{i,j}. $$

$$\text{DSS} = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} (u_{i,j} - v_{i,j})^2.$$  

(17)

1.3 Other processes in determination of cloud-top height

Before determination of cloud-top height, some image preprocesses must be done. These will strengthen the pattern recognition used in the match algorithm and decrease distortion of the images. These preprocesses include: (i) determination of effective overlapping area (effective stereo coverage of geo-synchronous satellite, accurate stereo height measurements may be made within this area); (ii) noise removal (by median or average filter); (iii) re-mapping (both satellite-W and satellite-E images must be remapped into the same coordinate system so that feature matching can be done with minimum distor-
tion); and (iv) normalization and enhancement (there is different brightness in satellite-W and satellite-E images because of different viewing angles. Standard techniques of image normalization and enhancement are used to process both images from the west and east satellites).

In addition, some post-processes must be performed after getting the distribution of cloud-top height. These post-processes include: (i) quality control on matching (removal of the "bad match" according to the assumption that cloud-top surfaces are continuous; sudden jumps in height are assumed to be produced by bad matches, matches with less match score (e.g. COEF < 0.5) are also considered to be bad); (ii) interpolating and smoothing of cloud-top height (some bad matches are removed during quality control on matching; the height is indefinite with bad match. Interpolation and smoothing of height is performed using neighborhood information surrounding the bad matches and the assumption that cloud-top surfaces are continuous); and (iii) the displaying of results (after getting the distributing of cloud-top height, we can make the 3-D quality view. Some visualization tools (PC-Vis5D, Cloud3D etc.) can be used to display the results. In addition, PC-Vis5D supports stereo view that enhances the visual experience by displaying images with true depth perception i.e. displaying simultaneously the left- and right-eye views as a "would-be human user" would see.

2 Tests and results

2.1 Making the stereo image pairs

Synthetic stereo image pairs were made from fictitious distributing image of cloud height and dual satellites with Eqs. (9) ~ (15). These stereo image pairs simulated for GMS-5 (140°E) and FY-2 (105°E) geo-synchronous satellites have been used to test the method of "height computations by cloud movement and sub-image matches" described before.

2.2 Cloud-top height computations

Fig. 3 shows the results of typhoon-cloud height computations, and Fig. 4 shows the results of front-cloud height computations. Some important parameters in computing are: \( \Delta h \) (height interval) = 0.25 km; the size of sub-image window on matching is \( 5 \times 5 \); COEF (correlation coefficient) > 0.5 is the quality control on matching.

![Fig. 3. Test and results for typhoon-cloud. (a) is the fictitious distributing image of cloud height (horizontal resolution is 1.25 km); (b) and (e) are stereo image pairs simulated from (a) for GMS-5 and FY-2 geo-synchronous satellites; (c) is the result of height computations with (b) and (e). In addition, (d) is the contour presentation of (a), and (f) is contour presentation of (c). Comparing (d) with (f), we found that the result images are very similar to the original fictitious images, and mean difference is about 500 m.](image-url)
3 Conclusions and future work

In this paper, accurate geometrical relationships of the satellites, the center of the earth, the projected-cloud and the true-cloud are presented. A new method of stereo height computations based on "cloud movement (height and position) and sub-image matches" is given. Some inspirer results have been made from the tests of synthetic stereo image pairs for GMS-5 and FY-2 geo-synchronous satellites.

In addition, the method of stereo cloud height measurements described above is independent of the spacecraft. It can be used in analyzing the data of stereoscopic observation for various spacecraft.

In our testing, only synthetic stereo image pairs were used to compute the stereo height. Real stereo pairs of GMS-5 and FY-2 will be processed later to study the performance of the algorithm. The method of stereo height computations is based on the terrestrial spheroid, and further research will be conducted based on the terrestrial ellipsoid in the future.

References