Solving liveness problem for marked nets by exhaustive coverability trees*

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Abstract It is well known that a coverability tree of a Petri net cannot solve reachability and liveness problems of the net because using symbol \( \omega \) (infinity) may lose some information. A solution to this problem is presented for a special kind of Petri net, marked net. With the combination of \( \omega \) and the increasing/decreasing information of token number, a new kind of coverability tree of marked nets, called exhaustive coverability tree (ECT), is proposed. It is shown with an example that an ECT can be used to detect deadlock.

Keywords: coverability trees, deadlock, marked nets.

The coverability tree (CT) method is one of the useful analysis methods for Petri net. However, reachability and liveness problems cannot be solved by a CT alone. Consider two place/transition nets (P/T-nets) \( N_1 \) (Fig. 1(a)) and \( N_2 \) (Fig. 1(b)). Though their structures and reachability sets are

![Diagram](image)

Fig. 1 Place/transition nets and corresponding coverability trees. (a) Petri net \( N_1 \) with a deadlock; (b) lived Petri net \( N_2 \); (c) ordinary coverability tree of both \( N_1 \) and \( N_2 \); (d) coverability tree of both \( N_1 \) and \( N_2 \) constructed in Yuan's way.

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different, they have the same CT (Fig. 1(c)). $N_1$ has a deadlock e.g. firing sequence $t_1, t_2, t_3$, while $N_2$ has not any deadlock. Such a problem results from the fact that for any integer $a$, $\omega + a = \omega$, i.e. $\omega$ cannot tell the increment or decrement of the number of tokens. For example, the number of tokens in $p_3$ decreases by 1 each time when $t_3$ is fired. Repeating firing $t_3$ will eventually result in a marking $M_1 = (0,1,0)$. In $N_1$, there is no transition enabled at $M_1$; but in $N_2$, $t_4$ is enabled at $M_1$. However, this cannot be reflected in their CTs.

Figure 1(c) is the CT of both $N_1$ and $N_2$ obtained by Yuan’s method, which can reveal the change of token number in a CT when using symbol $\omega^{[1]}$. In this method, in order to reflect the decreasing number of token in $p_3$ after firing $t_3$, the symbol $\omega_1(t_1) - \omega(t_3)$ is used to replace symbol $\omega$. However, this kind of CT still cannot distinguish the structure of $N_1$ and $N_2$ because it cannot extend the nodes containing symbol $\omega(t_1) - \omega(t_3)$. In this note a solution to this problem is presented for a subclass of P/T-nets, namely marked nets (MNs).

1 Exhaustive coverability tree

A marked net is also called an ordinary Petri net$^{[2]}$. It is such a P/T-net that all its arc weights are either 0 or 1$^{[3]}$. Many subclasses of P/T-nets such as marked graphs and free-choice nets are all considered as MNs.

The problem is solved by combining some information with $\omega$ as follows. Suppose that each node in a CT containing $\omega$ (with respect to place $s$) and the transition that yields this node is $t$.

(i) Mark $\omega^+1$ (abbreviated to $\omega^+$) instead of $\omega$ if a token is added to $s$ after firing $t$.

(ii) Mark $\omega^{-1}$ (abbreviated to $\omega^-$) instead of $\omega$ if a token is removed from $s$ after firing $t$.

(iii) Mark $\omega^0$ or simply $\omega$ if the number of tokens in place $s$ will not change after firing $t$.

This kind of coverability trees is called exhaustive coverability tree (ECT) in this note, because more details about the change of token number are included and a node containing $\omega$ can be extended exhaustively, and $\omega^{-1}$, $\omega^0$ and $\omega^+1$ have the same properties as $\omega$. So we call them $\omega$-components of a node in an ECT.

Algorithm (Constructing the ECT $G$ for $N$).

Let $N$ be a marked net with infinite capacities and let $M_0$ be its initiate marking. Let $W_i$ denote a vector $S \rightarrow \mathbb{Z}$ such that $W_i(s) = W(t,s) - W(s,t)$. Write $G_0, G_1, \cdots, G_i, \cdots$, where $G_i = (H_i, P_i)$, and $H_i$ is a set of nodes, $P_i$ is a set of arcs, $L_i$ is a set of arc labels, and let $P_i \subseteq H_i \times L_i \times H_i$ be a sequence of trees constructed according to the following steps.

1) Let $i = 0$, $H_0 = \{ M_0 \}$, $P_0 = \emptyset$, such that $G_0 = (\emptyset, \emptyset)$. Tag Mo “New”. Let $i = 1$, go to Step (2).

2) While “New” nodes exist in $G_i = (H_i, P_i)$, do the following steps.

2.1) Select a “New” node $E_i \in H_i$. 

(2.2) If \( E_i \in H_{i-1} \), then tag \( E_i \) "Dup" (meaning duplicate) and go back to (2.1) to select another "New" node.

(2.3) If there does not exist \( t \in T \), and \( t \) is enabled at \( E_i \); then tag \( E_i \) "Dead", and go back to (2.1) to select another "New" node.

(2.4) While there exists \( t \in T \), and \( t \) is enabled at \( E_i \), do the following.

(2.4.1) Add a new node \( E_{i,0} \). Note that \( E_{i,0} \) : \( S \rightarrow \{ N + \omega^{-1} + \omega^0 \} \) is a marking function, and \( E_{i,0}(s) \) is defined according to the following cases:

(a) if \( E_i(s) = \omega^m(m = -1, 0, +1) \), then \( E_{i,0}(s) = \omega^{m'}, m' = W_t(s) \);

(b) if \( E_i(s) \neq \omega^m(m = -1, 0, +1) \) and \( \exists E'_i \in H_i \) such that \( E'_i \leq E_i + W_t \), \( E'_i(s) < E_i(s) + W_t(s) \) and there exists a path in \( G_i \) from \( E'_i \) to \( E_i \), then \( E_{i,0}(s) = \omega^{-1} \);

(c) in other cases, \( E_{i,0}(s) = E(s) + W_t(s) \).

Tag \( E_{i,0} \) "New" and let \( H_i = H_i \cup \{ E_{i,0} \} \) and \( P_i = P_i \cup \{ (E_i, t, E_{i,0}) \} \).

(2.4.2) If \( \psi = \{ s \mid E_{i,0}(s) = \omega^{-1} \} \neq \emptyset \), let \( q = 1 \) and do the following.

While there exists a path \( p_i = h_0 t_0 h_1 \cdots h_r t_r h_{r+1} \cdots h_{n-1} t_{n-1} h_n \) from the root to \( E_i \) and there also exists \( \psi' \subseteq \psi \) satisfying the following conditions,

(a) \( \exists 1 \leq r \leq n \) such that \( \forall s'_1, s'_2 \in \psi' : h_r-1(s'_1) \) and \( h_r-1(s'_2) \) are not \( \omega \)-components and either (i) for all \( k = r, r + 1, \cdots, n, h_k(s'_1) = h_k(s'_2) = \omega^m(m_k = -1, 0, +1) \); or (ii) \( \forall s_{\psi'} \in \psi' : \exists r \leq j \leq n : h_j(s_{\psi'}) = \omega^+ \) and \( h_j(s) \neq \omega^+ \) for any place \( s \) different from \( s_{\psi'} \).

(b) There exists a node \( h_p \) in \( p_i \) such that \( \forall s \in \psi', s_2 \in \psi/\psi' : h_p(s_2) = \omega^{m_2}, h_p(s_1) \) is a natural number or \( h_p(s_1) = \omega^m, m_{s_1}, m_{s_2} = -1, 0, +1 \) and \( m_{s_1} < m_{s_2} \).

Then add a node \( E_{i,q} \) such that \( E_{i,q}(s_{\psi'}) = h_{r-1}(s_{\psi'}) \) for any \( s_{\psi'} \in \psi' \) and \( E_{i,q}(s_{\psi'}) = E_{i,q-1}(s_{\psi'}) \) for any \( s_{\psi'} \notin \psi' \). Tag \( E_{i,q} \) "New" and let \( H_i = H_i \cup \{ E_{i,q} \} \), \( P_i = P_i \cup \{ (E_i, t, E_{i,q}) \} \) and \( q = q + 1 \).

(2.5) Remove the tag "New" from \( E_i \). Let \( H_{i+1} = H_i, P_{i+1} = P_i, G_{i+1} = (H_{i+1}, P_{i+1}) \).

Let \( i = i + 1 \), repeat Step (2) until there is no "New" node in \( G_i \).

(3) \( G = G_i \).

Figure 2 shows an example of ECT construction by the algorithm. Fig. 2(c) is the ECT of the net shown in Fig. 2(a). Compared with its CT shown in Fig. 2(b), its ECT is rather complicated, but gives us more information about the change of markings of the net.

2 Detecting deadlock by exhaustive coverability tree

In this paper, deadlock is defined as a marking in which no transition is enabled.
Fig. 2  Marked net $N_1$ (a) and its coverability tree (b), and exhaustive coverability tree (c).

**Definition (Deadlock).** A marking $M \in M_0[$ of a P/T-net is called a deadlock if and only if $\forall t \in T : t$ is not enabled.

**Theorem**. Let $G = (H, P)$ be the ECT of some marked net $N$. $G$ has a deadlock if and only if $\exists E \in H : E$ is tagged “Dead”.

Go back to the problem presented at the beginning of this paper. Fig. 3(a) is the ECT of $N_1$ (Fig. 1(a)) and Fig. 3(b) is the ECT of $N_2$ (Fig. 1(b)). There are two nodes (representing the same marking $(0, 1, 0)$) tagged “Dead” in Fig. 3(a). This means that $N_1$ has a deadlock $(0, 1, 0)$. There is no deadlock in Fig. 3(b) since there is no node tagged “Dead”.

### 3 Conclusion

In this paper, a new kind of coverability tree for marked nets, called exhaustive coverability tree, has been proposed. This kind of coverability tree can reflect the increment and decrement of

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1) LU, Y. Q. Petri nets theories and their applications in telecommunications system, Ph. D. Dissertation, South China University of Technology, 1996.
the token number even though the symbol $\omega$ is used. It is shown that an ECT can be used to detect deadlock alone. However, an ECT is more complicated than an ordinary coverability tree of the same net. Explosion problems may arise when a marked net is complicated. Simplification techniques can be used to reduce the size of ordinary coverability trees\[4,5\].

References