Grid systems for geographic modelling and simulation: A review

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Abstract Geography requires a comprehensive understanding of both natural and human factors, as well as their interactions. Due to the complexity and multiplicity of geographic problems, various theories and methods for geographic modelling and simulation have been proposed. Currently, geography has entered an era in which quantitative analysis and modelling are essential for understanding the mechanisms of geographic processes. As the basic idea of quantitative spatial analysis, the specified space often needs to be partitioned by a series of small computational units (cells), i. e., grids. Thus, there is a close relationship between the grids and geographic modelling. This article reviews the mainstream and typical grids used for modelling and simulation. In addition to classification, the derived theories and technologies, including grid generation methods, data organization strategies, multi-dimensional querying methods, and grid adaptation techniques, are discussed. For integrated geographic simulation to explore comprehensive geographic problems, we argued that it is reasonable to build bridges among different types of grids (e. g., transformation strategies), and more powerful grids that can support multi-type of numerical computation are urgently needed.

Keywords Geographic modelling and simulation; Grid system; Grid classification; Grid construction; Numerical methods.

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1 Introduction

The real world is composed of diverse systems and subsystems, such as hydrologic, ecological, biological, and social systems. Although achievements have appeared in various fields of study, exploring the deep mechanisms of surface processes is still a persistent hot topic. With the evolution of geographic research from qualitative to quantitative methods, various numerical geographic analysis models, especially in the atmospheric and hydrologic domains, have been built to simulate and study geographic scenes and

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processes. These models employ different types of numerical methods, such as statistical methods, stochastic theories, artificial intelligence methods, and system dynamics methods.

When modelling, two types of strategies are mainly utilized to address geographic space. One is treating the experimental space as a whole (e.g., global statistics), while the other divides the space into various units to conduct more precise computations. For example, partial differential equations (PDEs), the lattice boltzmann method (LBM), and cell automata (CA) are the most commonly used methods to solve dynamic problems. Numerical methods controlled by PDEs, e.g., the finite difference method (FDM), the finite element method (FEM), and the finite volume method (FVM), are often applied when the computational domain is discretized with a grid system [1—4]. The LBM is an alternative discrete method that is often employed to simulate complex fluid systems and models a fluid consisting of fictive particles, and these particles perform consecutive propagation and collision processes over a discrete lattice grid [5, 6]. CA is also a discrete model that consists of a series of shaped cells or grids, but the edges and nodes of the grids are not normally considered. The new state of each cell is determined by the current state of the cell and the states of the neighbouring cells [7,8].

The above-mentioned methods indicate that grids are one of the most important and essential structures for numerical computations. As various types of grid systems have been designed for different simulation purposes, there is an urgent need to provide a better understanding of these systems to improve both models and simulations. The different types of grid systems need to be classified, and the theories and techniques derived from these systems, such as grid generation methods, data organization strategies, multi-dimensional querying methods, and grid adaptation techniques, should be discussed. Moreover, due to the multiple scales and varying complexity of geographical issues, a single grid system cannot be used to simulate complex geographic processes in many cases. Therefore, better solutions need to be found to employ existing grid systems for the more efficient reformation of the comprehensive geographical environments. This review article aims to address these issues as well as discuss potential strategies for using grid systems to support comprehensive geographic simulations.

2 Classification of grids

Gridding is the process of subdividing a region to be modelled into a set of units, e.g., small polygons or control volumes, which have regular or irregular shapes. The shapes of these units are one of the most obvious grid features. In general, these shapes mainly include triangles or quadrilaterals in 2D cases, and tetrahedrons, pyramids, tri-prisms and hexahedrons in 3D cases (as shown in Fig. 1).

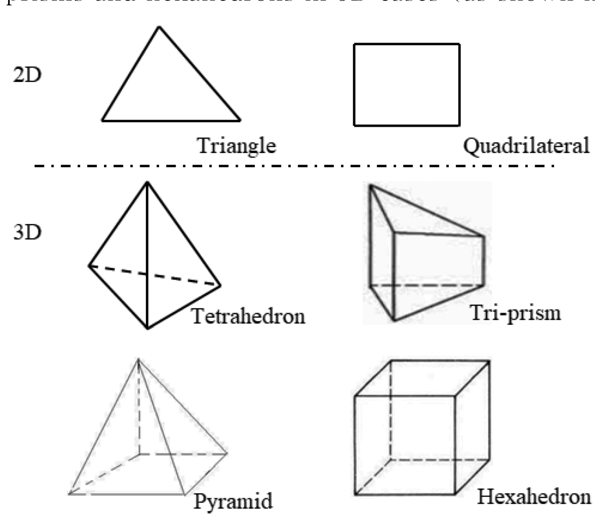


Figure 1 General grid shapes in 2D cases and 3D cases.

The values of one or more dependent flow variables (e. g. , velocity, pressure, and temperature) are associated with each unit. Normally, these variables represent values that have been averaged over different units. For approximate solutions to the conservation laws of mass, momentum, and energy, numerical methods are then used to compute more approximate values of the variables in each unit.

To describe grids as well as their units, the first step is to classify these grids into different categories according to different criteria. For example, grids can be classified as global grids or regional grids based on the spatial computational scales, visualization-oriented grids or modelling-oriented grids according to different application objectives, and two-dimensional (2D) grids, surface grids and three-dimensional (3D) grids based on the spatial dimensions. As the structure of a grid is most closely related to the corresponding numerical method [9], among these classification methods, the structure of the grid is normally employed as the most important criterion. According to the structure of the grid, discrete grids can currently be classified into four main categories: structured grids, un-structured grids, hybrid grids and Chimera grids. Table 1 provides an overview of the characteristics of different categories of grids classified by structure. These characteristics are summarized in normal but not absolute cases.

Table 1 Characteristics of four types of grids classified by structure

	Structured grids	Unstructured grids	Hybrid grids	Chimera grids
Number of neighbours	Fixed	Varied	Varied	Fixed or varied
Grid shape	Rectangle Quadrilateral Hexahedron	Triangle Quadrilateral Tetrahedron Hexahedron	Triangle Quadrilateral Tetrahedron Hexahedron	Triangle Quadrilateral Tetrahedron Hexahedron
Grid coding	Easy	Normal	Difficult	Difficult
Boundaryfitting	Feasible	Desirable	Good	Desirable
Numerical modelling	Easy	Normal	Difficult	Difficult
Usage	Visualization Data organization Simulation	Visualization Data organization Simulation	Simulation	Simulation
Application fields	Fluid dynamics Solid dynamics	Fluid dynamics Solid dynamics	Fluid dynamics Solid dynamics	Fluid dynamics Solid dynamics

2. 1 Structured grids

Structured grids have one main feature, i. e. , the arrangement layout is ordered both vertically and horizontally. For each unit in a grid, the number of its first-order neighbours is fixed, and it is relatively easy to implement grid coding and discrete computational programming. If the orthogonality of the grid is considered, structured grids can be further divided into orthogonal curvilinear grids and non-orthogonal curvilinear grids [10]. The main advantage of orthogonal grids is that the formulas of the equations that control the model dynamics can be expressed in a more concise way under an orthogonal coordinate system. If the FDM, which is more often used in numerical simulation, is implemented on structured grids, especially orthogonal grids, the main differential operators in the PDEs can be discretized intuitively and easily. Based on the spatial dimension, structured grids can be divided into 2D structured grids, surface structured grids and 3D structured grids.

2. 1. 1 Two-dimensional (2D) structured grids

In 2D cases, structured grids mainly include general curvilinear grids, and structured rectangular grids (e. g. , orthogonal curvilinear grids and conventional longitude-latitude coordinate grids, as shown in Fig. 2). These types of grids are often utilized in regional hydrology or hydrodynamic modelling [11—15].

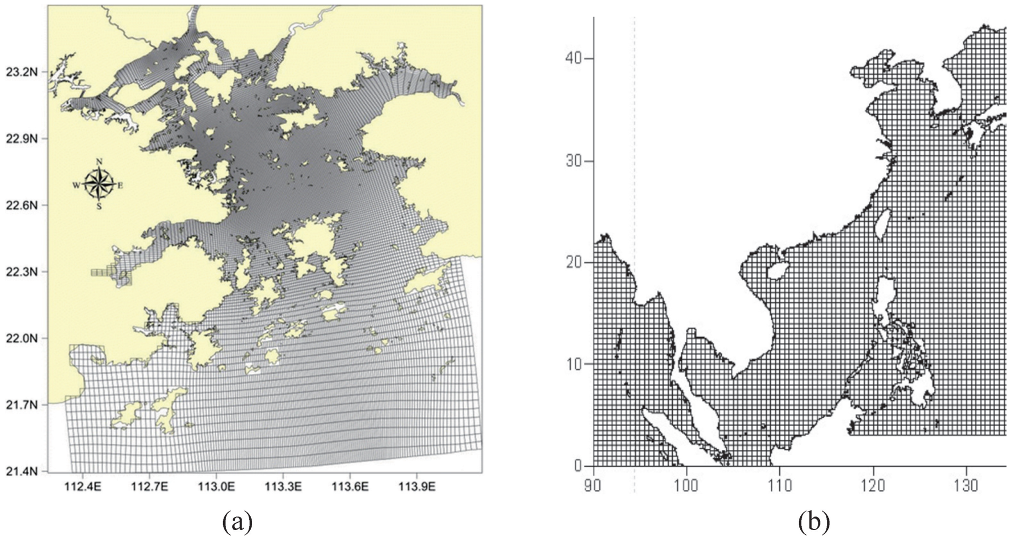


Figure 2 Grids for 2D cases of regional ocean modelling. (a) The orthogonal curvilinear grids for the Pearl River estuary [14]; (b) the longitude-latitude grids for the marginal seas in China.

2.1.2 Surface structured grids

A typical type of surface structured grid is a spherical grid, which has widespread applications in global numerical weather and marine forecasts [9,16,17]. These spherical grids should meet some conditions before being used in models and simulations [18,19]. Currently, spherical grids include different types of grids (see Fig. 3), such as traditional longitude-latitude (LL) grids [20], reduced LL grids [21], general orthogonal curvilinear grids [22], spherical structured quadrilateral grids based on cubes [23—25], and spherical rhombus grids based on icosahedrons [26].

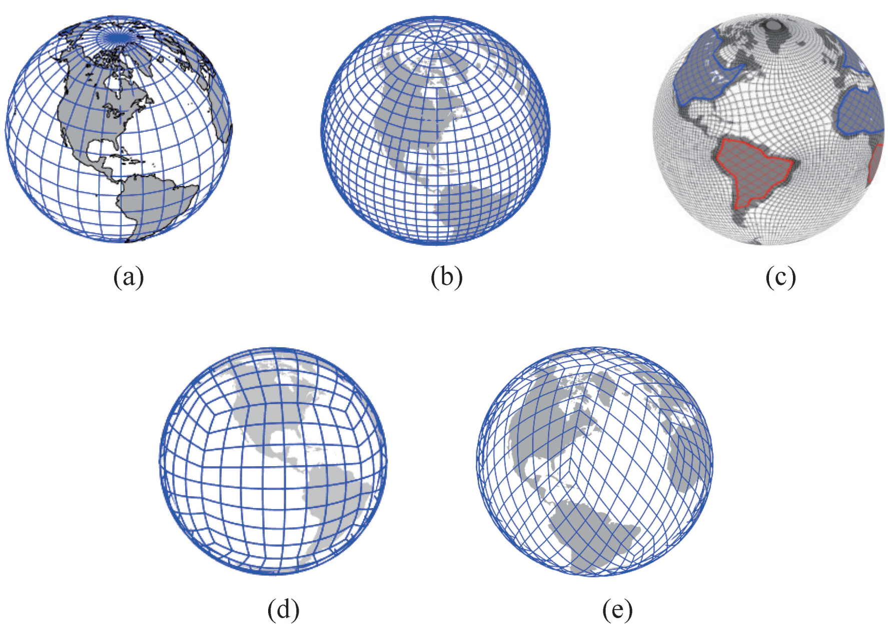


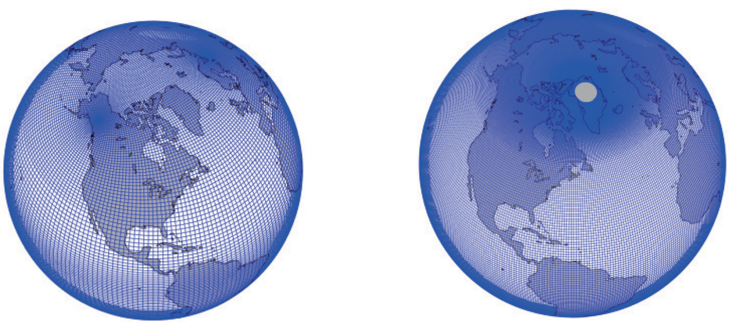
Figure 3 Structured grids on a spherical surface. (a) Traditional LL grid; (b) reduced LL grid; (c) general orthogonal curvilinear grid (modified by Xu et al., 2015); (d) cubed sphere grid; (e) icosahedron sphere rhombus grid.

The traditional LL grid is one of the most widely used surface structured grids. However, the traditional LL grid has a defect, as most of the global atmosphere or ocean numerical models suffer from issues related to the "pole problem" that occurs in the areas adjacent to the two poles on the sphere. This problem could

be relieved by several approaches, including filtering [27, 28], semi-Lagrangian and semi-implicit integration [29,30], transformation of the coordinate system [31], and grid topology improvement [32]. To avoid this problem, many global ocean numerical models based on LL grids were built without considering the Arctic Ocean [33—37].

Reduced LL grids are improvements of traditional LL grids [38]. The key idea behind reduced LL grids is to properly merge grid units along certain latitudes near the poles, which greatly relieves the "pole problem".

Spherical orthogonal curve grids are commonly used in regional ocean models. This type of grid can fit the coastline better than general structured grids. Usually, the generation of these grids relies on numerically solving elliptic PDEs; other restrictions can be imposed on this kind of grid if necessary, such as grid smoothness or quasi-uniformity of grid units [39]. Specifically, to solve the "pole problem", special kinds of orthogonal grids have been employed by many global ocean numerical models (see Fig. 4). For example, displaced north pole grids [40—42], and multipole grids generated by the conformal mapping method [22,43,44] were adopted by the parallel ocean and ice model (POIM), parallel ocean program (POP) and modular ocean model (MOM). The core idea of such grids is to transfer the north pole to an adjacent land or beyond the area of interest. The two poles in the traditional LL grid could be moved to any other region on the sphere by using the conformal mapping method [45]. However, it is difficult to perform grid refinement locally on these grids.



(a) (b)

Figure 4 Spherical orthogonal curvilinear grids. (a) Tripolar grid; (b) displaced North Pole grid.

2.1.3 Three-dimensional (3D) structured grids

In 3D cases, hexahedral grids are commonly employed in many critical applications that require volumetric PDEs to be solved. The use of this grid type is mostly due to the naturally embedded tensor product structure, large tolerance for anisotropy and lower numerical stiffness of hexahedral grids compared to that of unstructured meshes (e.g., tetrahedral meshes) in the numerical computation aspect.

Based on the above-mentioned merits, 3D structured grids can be employed to compute different targets. For example, 3D cubed sphere grids (see Fig. 5) are normally applied to simulate and model the dynamics of Earth's mantle and magnetohydrodynamics (MHD) flow problems, such as solar wind modelling [46—49], and 3D hexahedral grids (see Fig. 6) are normally used to compute the near-field aerodynamics of 3D solid geometry [50,51].

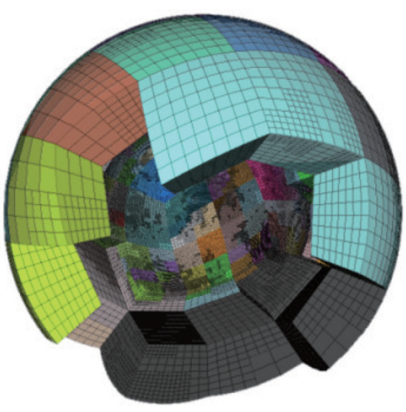


Figure 5 The cubed sphere shell grids with local refinement [46].

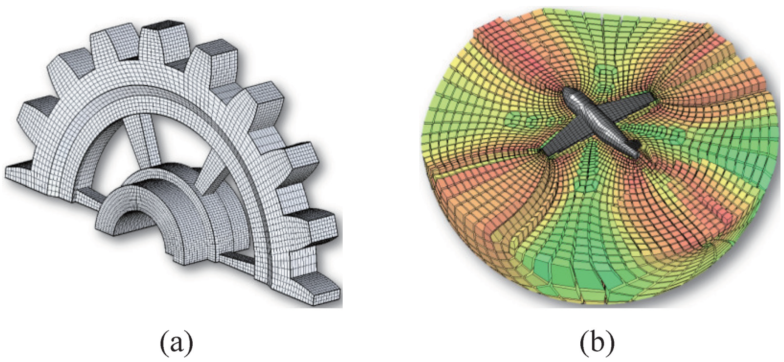


Figure 6 Two cases of structured hexahedral grids (https://www.lacan.upc.edu/ruiz). (a) A toothed gear; (b) space around an airplane.

2.2 Unstructured grids

Unstructured grids are mainly composed of arbitrary units. The main advantage of unstructured grids is their flexibility to complex geometry. For unstructured grids, the nodes and units can be distributed more freely than those in other grids, which better fits the boundary of the computation domain. Unstructured grids adopt a random data structure to facilitate grid adaptation in regions of interest. However, unstructured grids also have their own drawbacks, i.e., under the same conditions, the number of nodes or units may be more than those in a structured grid. In addition, it is more difficult to implement numerical methods using unstructured grids.

2.2.1 Two-dimensional unstructured grids

In 2D cases, the unstructured grid category mainly includes triangle grids (see Fig. 7(a)) and unstructured quadrilateral grids (see Fig. 7(b)). In general, 2D unstructured grids are widely used in regional ocean models, especially if the boundary of the computation domain has an irregular shape [52—56].

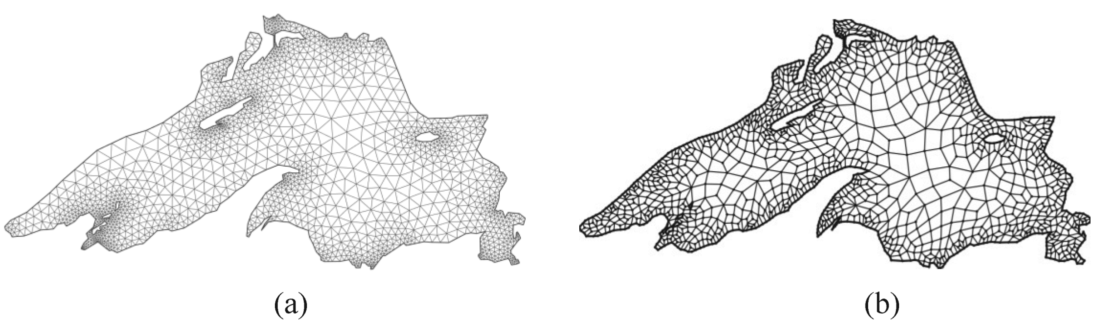


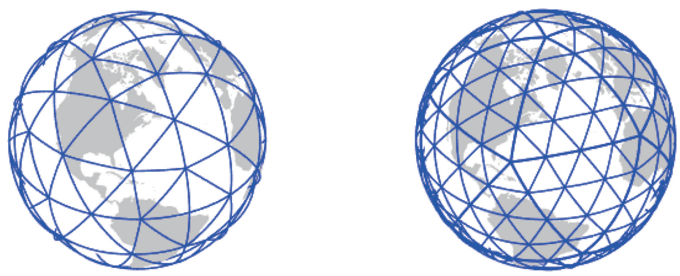
Figure 7 Unstructured grids for Lake Superior. (a) Triangles; (b) quadrilaterals [57].

A typical usage of unstructured triangle grids is the development of the Finite Volume Community Ocean Model (FVCOM), which is a famous oceanographic numerical model that was developed by the Marine Research Group of the Massachusetts Institute of Technology. FVCOM was developed based on the FVM with unstructured grids. Later, the AO-FVCOM module was developed using the stereographic projection method [58]. By incorporating AO-FVCOM, the FVCOM has the ability to simulate tidal dynamics in the global ocean [59]. However, there is a prerequisite for the triangle mesh used by the FVCOM, i.e., there must be only one grid unit site at the North Pole.

Other alternatives have also been utilized for regional or global ocean modelling, i.e., unstructured quadrilateral grids, such as the grids employed by the spectral element ocean model (SEOM) [60, 61]. However, compared to triangle grids, quadrilateral grids have less flexibility regarding the geometry of a specified computation domain. Therefore, quadrilateral grids are not as popular as triangle grids.

2.2.2 Unstructured surface grids

Some unstructured surface grids are used for surface visualization, some are used for organization of global spatial data [62—65], and some are used for simulation, especially those on spherical surfaces. Over the last two decades, regular octahedron and icosahedron (see Fig. 8) grids were rapidly developed based on a regular octahedron. Goodchild used a recursive iterative decomposition method to establish a spherical triangulation [66]; later, White used five different projection methods to compare some properties of a spherical triangulation based on a regular octahedron and a regular icosahedron and found that the icosahedron spherical grids were more desirable [67].



(a) (b)

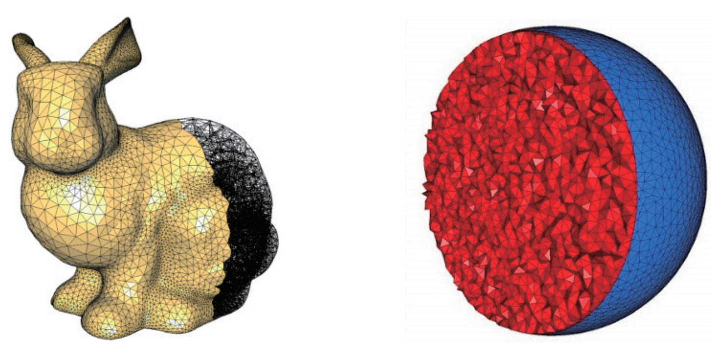
Figure 8 Unstructured spherical grids based on (a) octahedrons; (b) icosahedrons.

The face of each unit of an icosahedron spherical grid is represented using a triangle, and the angles and metrics of the grids are relatively uniform. These types of grids are used not only for global atmosphere numerical models [68—73] but also for global ocean numerical models [74—76]. However, the data operations on this type of grid require an index, and a topological relation table should be established for each unit in advance. Moreover, numerical methods built on such grids are usually very complicated.

2.2.3 Three-dimensional unstructured grids

In 3D cases, unstructured grids mainly include three basic unit types: tetrahedrons, tri-prisms, and pyramids.

These grids are employed in many applications to solve volumetric PDEs. Three-dimensional unstructured grids are more flexible than structured grids (e.g., hexahedral grids) when modelling complex geometry domains with little distortion of mesh. For the sphere shells in 3D cases, the unstructured grids are mainly based on tri-prisms, which are normally used to simulate 3D mantle convection and Sun-solar wind systems [77,78]. In 3D visualization or modelling, tetrahedral grids are preferred to model 3D objects with complex geometries (see Fig. 9).



(a) (b)

Figure 9 Examples for tetrahedral grids. (a) Stanford bunny [79]; (b) a solid sphere (https://doc.cgal.org/latest/Mesh_3/index.html).

2.3 Hybrid grids

The core idea of hybrid grids is to adopt different types of grids in different sub-regions based on the domain decomposition methods (DDM). Hybrid grids can be classified into two main categories, which are structured-unstructured mixed grids, and unstructured mixed grids.

In the 2D case, the hybrid grids are often used when the geometry of the computation domain is complex [80,81]. In general, the structured grid part is used for sub-regions with regular boundaries; and the unstructured grid part is used in areas with irregular boundaries (see Fig. 10).

For surface hybrid grids, spheroid degenerated octree grids (SDOGs) have received much attention (see Fig. 11) [82]. Undoubtedly, such grids have advantages in global data visualization, but due to different types of grids in different sub-regions, special sewing techniques are required for the common boundaries shared by two sub-regions adjacent to each other, which increases the complexity of the topological relationships and the probability of cracks.

In the 3D case, hybrid grids are used in many fields, especially in the optimization design of aerodynamic or hydrodynamic configurations [83].

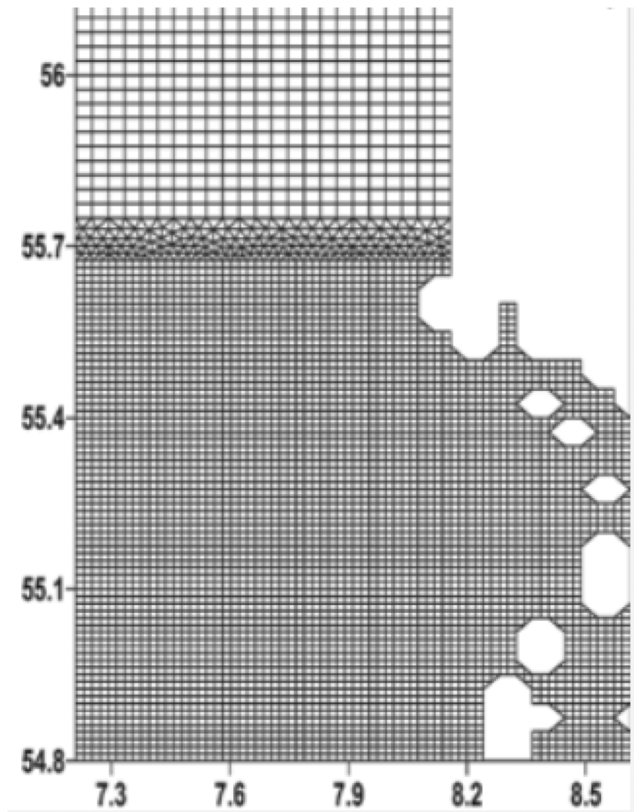


Figure 10 An example of a hybrid grid in a 2D case (Danilov et al., 2015).

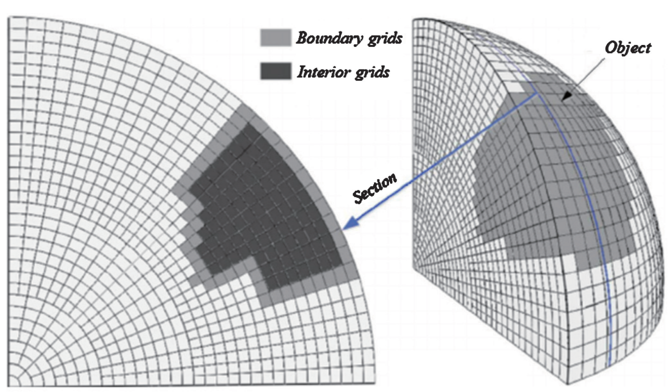


Figure 11 An example of a SDOG; the boundary grids and interior grids of a spatial object (Yu et al., 2012).

However, there is one main drawback with hybrid grids. In most cases, different types of grids, different sets of equations and different numerical schemes are used in the two sub-regions adjacent to each other. Therefore, special care must be taken to address data mapping/matching across the shared grid boundaries.

2.4 Chimera grids

Chimera grids were first proposed in the 1980s [84]. Chimera grids compose a set of overlapping structured grids, which are independently generated and body-fitted, yielding high-quality grids that are

readily accessible for efficient solution schemes [85]. Therefore, Chimera grids are also called "overset grids". Chimera grids have shown their ability to support computation on complex geometries.

In 2D cases, Chimera grids are usually employed to solve problems with multiple irregular-shaped blocks placed in the computation domain; a regular Cartesian grid is often chosen for the background grid, and body-fitted mesh is used around the blocks with the irregular shapes [86, 87]. An example of a Chimera grid can be seen in Fig. 12.

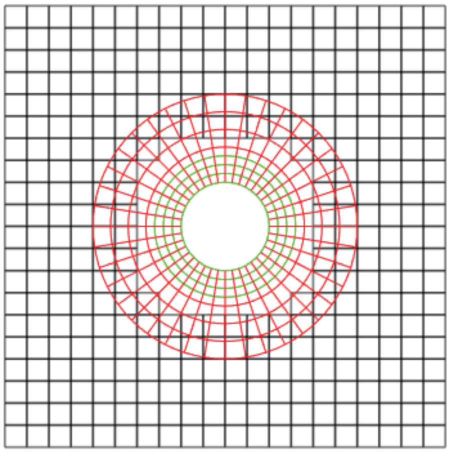


Figure 12 Cropped view of a Chimera grid, showing the boundary layer grid (green), transition grid (red) and background grid (black).

A typical surface Chimera grid is a Yin-Yang grid. Fig. 13 shows several kinds of Yin-Yang grids, including Yin-Yang grids on spherical surfaces [88, 89], Yin-Yang grids on spherical shells, and Yin-Yang-Zhong grids for spheroids [90—92].

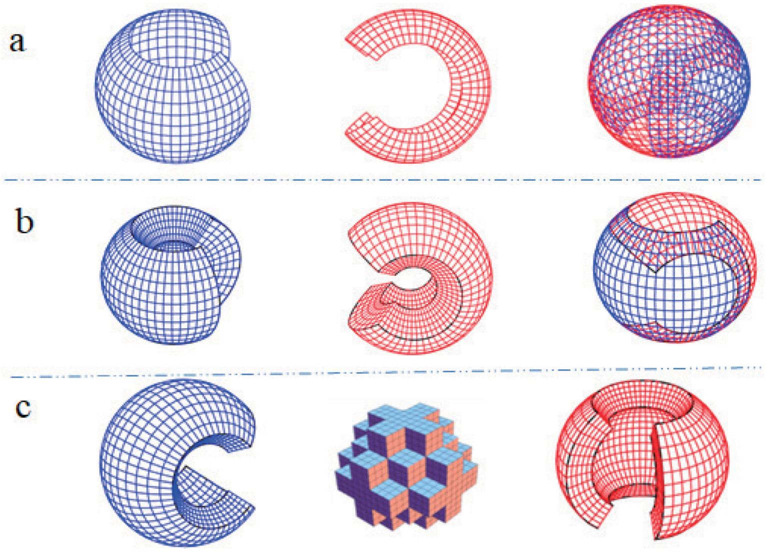


Figure 13 Yin-Yang overset grids. (a) on a spherical surface; (b) for a spherical shell; (c) Yin-Yang-Zhong grid for a spheroid [92].

In 3D cases, Chimera grids are also widely used in many other fields, especially in the optimization design of airplane or watercraft structures [93,94] (see Fig. 14).

Despite their geometrical flexibility, Chimera grids have a drawback, i. e., interpolation of data in the overlapping regions is a tricky problem. It is difficult to design a suitable numerical method that conserves the physical variable.

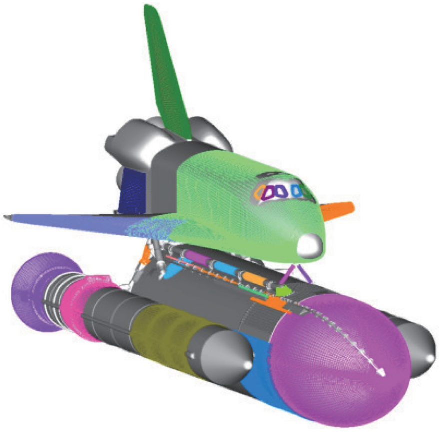


Figure 14 Chimera grids for the complex surface of an aircraft, from OVERGRID Version 2. 3 (https://www.nas.nasa.gov/publications/software/docs/chimera/index.html).

3 Grid-derived theories and techniques

For a deeper analysis and to explore the theories and techniques derived for grids, it is first necessary to clarify the basic components of a grid unit (see Fig. 15). For 2D cases, a grid unit is composed of the node/vertex, the edge and the face; for 3D cases, in addition to the former three components, the volume should be included.

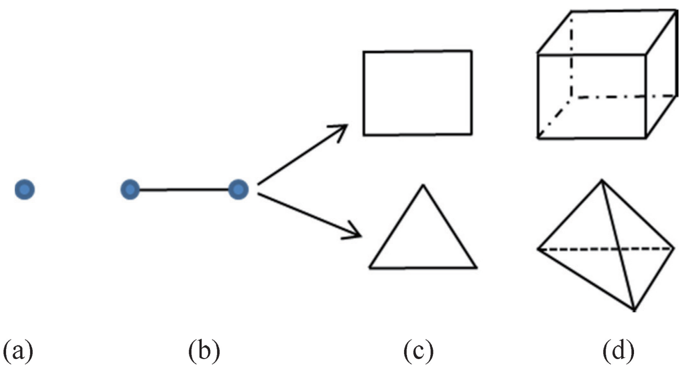


Figure 15 Grid components in 3D cases. (a) Node; (b) edge; (c) face; (d) volume.

3.1 Grid generation

3.1.1 Generation of structured grids

The generation of structured grids is mainly based on algebraic methods, such as linear interpolation or the transfinite interpolation method. In 2D cases, the methods for the generation of body-fitted grids or orthogonal curvilinear grids are relatively tricky, including conformal mapping, elliptic PDEs, and the variation method. If structured surface grids can be parameterized in a 2D plane, then the grids are usually generated in that parameter domain. More generally, based on DDM, the surface could be approximated by a series of small patches with relatively regular shapes, and each patch can be handled in a 2D local coordinate system by mapping. In 3D cases, octree techniques are useful for the generation of structured grids, and alternative methods include advancing fronts, sweeping, paving, and plastering.

There are various software tools accessible for the generation of structured grids, especially in 2D cases. Generators for orthogonal curvilinear grids include SeaGrid [95], Deft3D [96], and Gridgen [97], while the G-Cubed toolkit for tripolar grids can be used to generate tripolar grids for spherical surfaces. In 3D cases, ICEMCFD (the Integrated Computer Engineering and Manufacturing code for Computational Fluid Dynamics) and CUBIT [98] may be helpful and make the procedure for generating structured grids more efficient.

3.1.2 Generation of unstructured grids

In 2D cases, the methods commonly used for triangle mesh generation include the Delaunay method and the advancing-front method (AFM) [99,100]. By means of domain decomposition or coordinate mapping, these methods could also be applied to a surface. For instance, based on the Delaunay method or AFM, spherical triangle grids could be generated, which could then be used to build numerical ocean models [101—104]. The key idea of the above methods can also be extended to the generation of unstructured quadrilateral grids [105], while other generation methods include Q-TRAN methods and Blossom-Quad methods [106—108].

For unstructured grids on a general surface, if they could be parameterized in 2D planes, then the generation of this type of grid can be also generated by DDM, and the surface can be approximated by a series of small patches with relatively regular shapes. In addition, matching cubes or matching tetrahedrons are commonly used to generate triangle meshes on implicit surfaces.

For unstructured grids in 3D cases, the Delaunay-based method is normally used to generate tetrahedral meshes [109,110].

There are various commercial or open-source software packages for the generation of unstructured grids, and software with more powerful functions may integrate numerical models within itself. In 2D cases, generators for triangle grids include EasyMesh [111], Triangle [112], MESH2D, DistMesh [113], and Surfacewater Modeling System (SMS); generators, for unstructured quadrilateral grids include QUAD_MESH, QUAD-GEN, and AUTOMESH-2D [114]. In 3D cases, generators for tetrahedral grids include TetGen [110], Computational Geometry Algorithms Library (CGAL) [115], and Gmsh [116]. These tools can be helpful and make the generation procedure for unstructured grids more efficient.

3.1.3 Generation of hybrid grids and Chimera grids

There are some similarities between the generation of hybrid grids and Chimera grids. Based on DDM, complex geometry objects can be divided into multiple sub-regions with relatively simple geometric boundaries. Then, grids in each sub-region can be generated independently [83,117—119]. The major difference is that for Chimera grids, there is a prerequisite that two sub-regions should overlap each other if they are adjacent.

In 3D cases, the generation of Chimera grids is normally composed of the following four steps: extraction of geometric shapes, generation of surface grids, generation of volumetric grids, and interpolation of the relationships between grids in overlapping regions. The first three steps can be implemented by existing universal 3D software, such as ANSYS/Pointwise [120], ABAQUS [121], Gambit, EAGLEView [122], and Interactive Geometry Modeler and Multi-Block Structured Grid Generator (IGG). The process of establishing interpolated relations between grids in adjacent sub-regions is the core of Chimera grids, and this process requires three key technologies: overlap of wall grids, hole-cutting and identification of interpolation stencils. These technologies are essential to overset grids. Alternative programs and software systems for Chimera grids include PEGASUS5 [123], SUGGAR [124], OVERGRID, and Chimera Grid Tools (CGT).

3.2 Data organization strategies

Grids should be organized in a suitable data structure; then, operations such as searching could be performed in an optimal way. In general, the commonly used data structures include linear lists, linked lists, hash tables, heaps, and tree structures; these structures are widely employed to organize data in grids and for other operations.

Among these data structures, tree structures are popular if local grid refinement is needed, and there are several types of tree structures, including binary trees, quad-trees and octrees.

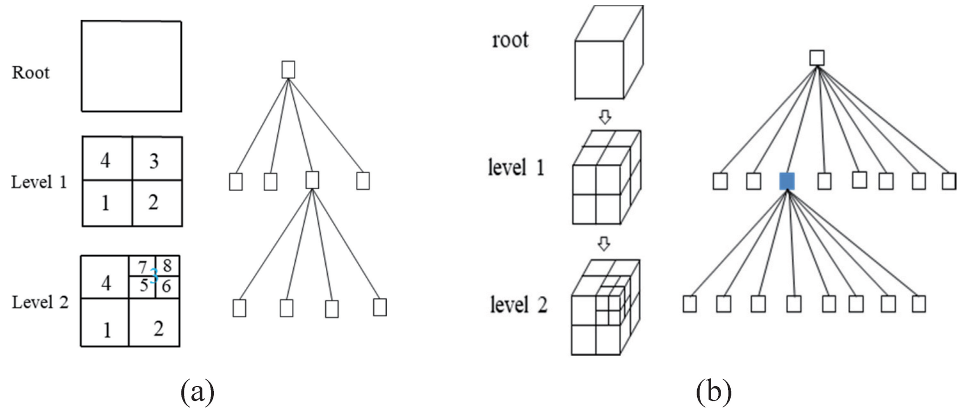


Figure 16 The grid refinement and corresponding data structure. (a) Quad-tree refinement; (b) Octree refinement.

For Cartesian grids in 2D cases, the quadtree data structure is very efficient if local refinement is involved [125,126]; similarly, the octree structure is suitable for grid refinement in 3D cases [127,128], as shown in Fig. 16. Tree structures can also be employed to generate unstructured grids, such as triangle or tetrahedron grids. For triangle grids, the binary tree structure can be regarded as another alternative used to organize grid data (see Fig. 17).

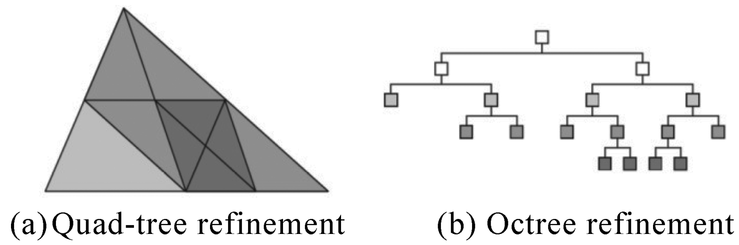


Figure 17 The triangle refinement and binary tree structure [129].

3.3 Multi-dimensional querying methods

Most models used for the simulation of geographical processes are computation-intensive and require extensive runtimes. To attain an efficient numerical simulation or parallelization, the data structures are well suited for optimization if they are in vector-like shapes. In addition, data locality should be maintained when mapped from d-dimensional ($d>=2$) grids to one-dimensional vectors. A common technique is data-reordering, which includes the Cuthill-McKee algorithm (CM), the Reverse Cuthill-McKee algorithm (RCM), and minimum degree ordering [130,131].

In most cases, a very efficient and even better reordering mechanism stems from space-filling curves (SFCs). An SFC that is well suited for structured quadrilateral grids is the Hilbert curve or Z-order curve; an SFC that is well suited for triangular bisection grids is the Sierpinski curve [132—136] (see Fig. 18).

There is less literature on 3D cases due to the immaturity of the methods and techniques, and further studies are required in this aspect [137,138].

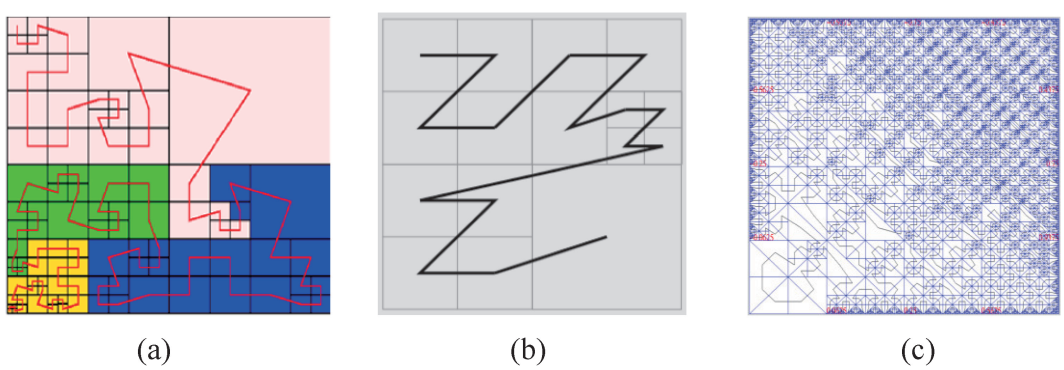


Figure 18 The SFC curves. (a) Hilbert curve for quadtree grids [135]; (b) Z-order curve for quadtree grids [139]; (c) Sierpinski curve for adaptive triangle grids refinement [136].

3.4 Grid adaptation techniques

In many cases, to achieve equilibrium between the computation load and the efficiency, it is necessary to perform grid adaptation during simulations or modelling.

Grids with variable resolutions have been widely used for atmospheric modelling [140—145], and ocean modelling [146—151] at both regional and global scales. In particular, there are three primary techniques, including grid nesting, grid stretching, and adaptive mesh refinement (AMR) techniques [32] (see Fig. 19). The first two techniques can also be called "R-refinement", and the third is called "H-refinement".

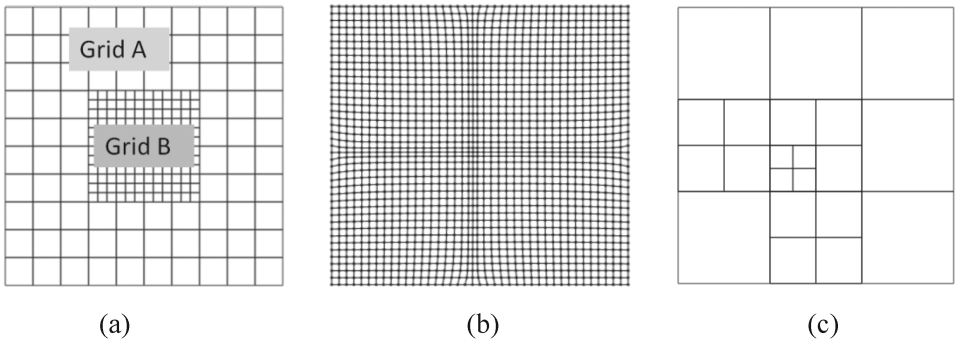


Figure 19 Three major techniques for regular grid adaptation in 2D cases. (a) Nested grid; (b) stretched grid; (c) AMR grid.

Grid nesting is widely used for regional atmosphere or marine forecasts and to downscale large-scale or global simulations [152—157]. Grid nesting can be employed to combine large-scale simulations with realistic mesoscale forecasts for specified domains. For instance, the well-known regional climate models, such as MM5 (the Fifth-Generation Penn State/NCAR Mesoscale Model), WRF (the limited-area Weather and Research Forecasting Model) [158] and CRCM (the Canadian Regional Climate Model) [159]. Typically, the grid resolution ratios between the refined grids and the coarse grids should not be more than 5 [160], then a block of fixed-size refined grids is embedded in a coarse-resolution model, which provides background information fields for the lateral boundary conditions of the nested subdomains. In general, the simulation is performed by different models in the two domains with different resolutions. Therefore, special techniques must be taken to minimize the consequent numerical inconsistencies across the fine-coarse grid boundaries. There are mainly two techniques: one-way nesting or two-way nesting [161]. For the former, the solution on the coarse grids is independent of that on nested grids, i. e. , the solution on the nested area has nothing to do with the solution on the coarse grids, and the boundary conditions of the nested sub-regions are completely controlled by the solution on coarse grids. For the latter, the solution on the coarse grids is continually updated by that on the transition/halo region where the refined grids and coarse grids coincide. In most cases, the two-way nesting is superior to one-way nesting.

Grid stretching is another technique used to increase the resolution in local areas while keeping the total number of grid nodes or cells constant [141,162—164]. Unlike the nested technique, models based on stretched grids do not require interpolation to make the data consistent between the coarse-resolution grids and the fine-resolution grids.

For process simulations, dynamic AMR is the most flexible variable-resolution technique. The goals of AMR are to refine grids locally that need finer grid resolution and coarsen the grids if a finer resolution is no longer required, which can readily vary the number of grid points as demanded by the adaptation criteria [165—169].

There are two main strategies for AMR in 2D Cartesian grids, which are non-conforming refinement and conforming refinement. The first strategy produces non-conforming grids, and during the subdivision process, every "parent cell" is divided into several "child cells". More specifically, for every parent cell, a new point is added to each edge centre. For 2D structured quadrilaterals, a new point is also added at the cell centroid, and four new "child cells" would be produced by joining these points. Thus, each quadrilateral parent gives rise to four new offspring. The advantage of such procedure is that the overall topology remains the same (with the child cells taking the place of the parent cell). The subdivision process is similar for a triangular parent cell, as shown in Fig. 20(a). In such case, a quadtree data

structure is very efficient for grid management. The second strategy produces quadrilateral grids with conforming variable resolution (see Fig. 20(b)).

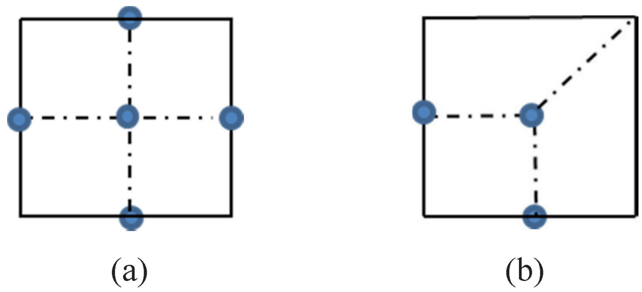


Figure 20 Principle strategies for local refinement of quadrilateral grids. (a) Non-conforming refinement; (b) conforming refinement.

There are three main strategies to implement local grid refinement for AMR used in triangle mesh, including longest edge/side (LE) bisection, barycentric partition, and 4T similar partition (as shown in Fig. 21). The most simple approach is the two triangles longest edge (2T-LE) bisection method [170]; based on these three strategies, there are still some other alternative AMR strategies for triangle meshes [171—174].

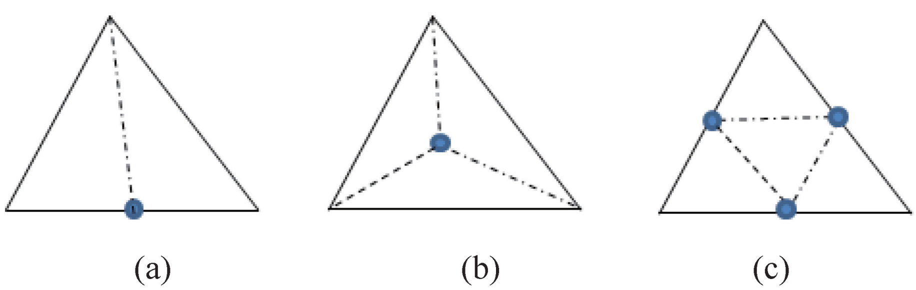


Figure 21 Principle strategies for the local grid refinement of triangles. (a) LE bisection; (b) barycentric partition; (c) 4T similar partition.

Hexahedrons and tetrahedrons are the most commonly used units in 3D cases. For hexahedron refinement, there are four primary refinement templates (see Fig. 22) [175—181]. For partitioning a tetrahedron, there are several approaches; one of the most commonly used techniques is the longest edge (LE) bisection (See Fig. 23) [182], and other alternatives include the 8T-LE, standard partition and 3D barycentric partition [183] (see Fig. 24).

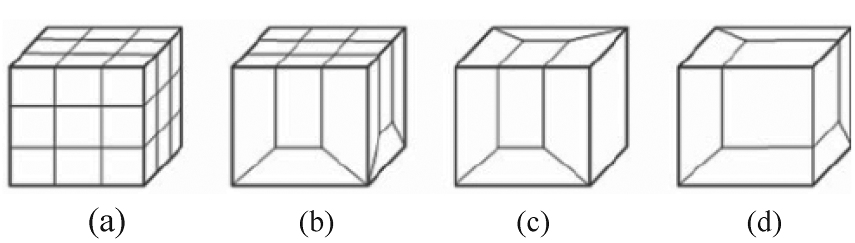


Figure 22 Templates of hexahedral grid refinement [177]. (a) All-refinement template; (b) face-refinement template; (c) edge-refinement template; (d) point-refinement template.

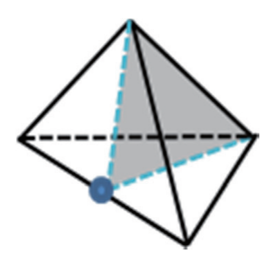


Figure 23 The LE bisection of a tetrahedron.

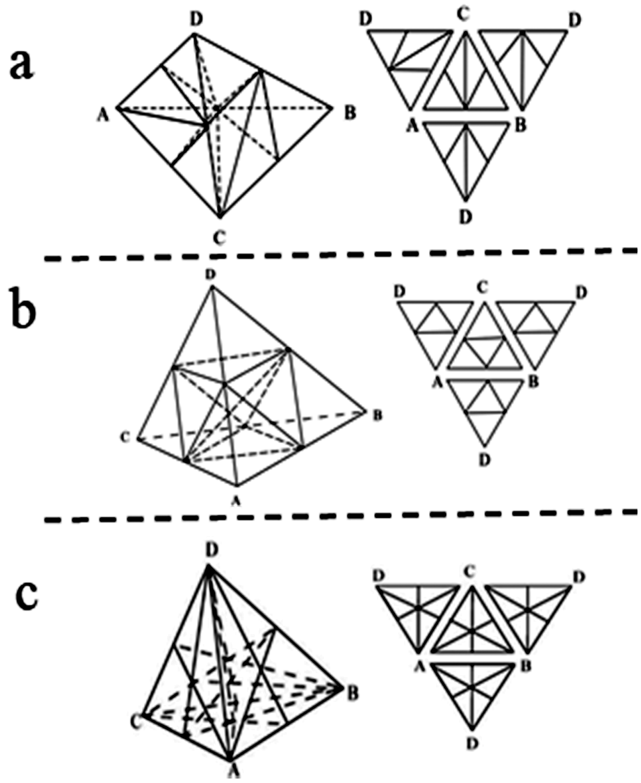


Figure 24 Principle techniques for local refinement of tetrahedron grids [183,184]. (a) 8T-LE; (b) standard partition; (c) barycentric partition.

4 The existing issues and settlements

Despite the great progress of the methods and techniques used in geographic modelling and simulation, a series of challenges related to many aspects remains.

4.1 Challenges in grid construction

In recent years, quantitative geography has rapidly developed, and more objects to be modelled and phenomena to be simulated have been identified. Grids are the basic units for these implementations. Due to the immaturity of the theories and techniques used for the generation of grids, it is still challenging to generate high-quality grids for sophisticated geometry [185]. For unstructured grids, it is still difficult to construct numerical methods with high-order accuracy [186]. For hybrid/mixed and Chimera grids, more effective/favourable methods are needed to match the data between grids across the common boundaries of adjacent sub-regions and design a conservative numerical method [187].

In geo-modelling or simulation, many problems are computation-intensive, which require extensive computer resources and runtimes [16]. Moreover, many data-management-based grids cannot be used directly for simulation because the interaction mechanisms between the grid nodes and edges in those grids are often ignored. It is necessary to develop more efficient methods and techniques for grids generations and queries.

4.2 Challenges in coupling grids for comprehensive modelling

During modern geography research, it has become clear that a single model alone has very limited capacity to simulate complex geographical phenomena. The trend of coupling multiple models to model a specified complex system has become increasingly popular [188—190]. It is probable that these models were built on different types of grids.

There are many different types of grids used in geographical modelling, and it is significant to manage and manipulate the data in an efficient way. There are two main strategies: using only a single category of

grid to model all geographic processes and building bridges among various types of meshes used by many different geographic models. The first method sounds ideal and desirable, but in practice, it is currently difficult to accomplish this goal because of the complexity and multiple scales of geographic processes. Many resources regarding various grids and models have been built by predecessors, and these valuable resources should not be ignored and left unused. Therefore, it is reasonable and feasible to build bridges among different types of grids and created couplers among different models. This method is effective, and many achievements have been made by using this approach.

5 Summary

This article reviewed the main categories of grids used for numerical simulation models, as well as the related theories and techniques, including grid generation, data organization strategies, multi-dimensional grid querying methods, and grid adaptation techniques. Existing challenges and potential solutions were also discussed.

It is obvious that with the development of quantitative geography, various models have been applied for realistic geometric problems, and grids have become increasingly important. The grids summarized in this paper are only part of the existing grids, and related technologies are not fully demonstrated, but grid systems are undoubtedly an important research direction. Numerical modelling based on 2D planes to general surfaces and then to 3D, or even higher-dimensional space would not work without the support of grids. There is an urgent need to deepen the existing research on grids. In the future, grid structures, operation efficiencies, and the generation of complex grid geometries or grids to support coupled modelling should be explored as further development.

Acknowledgments

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